20. SYMMETRY AND ITS GENERALIZATION


Three equal, square prisms with axes along [010], [001] and [010] intersect in a cube if the prism faces are parallel to the (010), (001) and (100) planes. They intersect in a rhombic dodecahedron if the prism faces make angles of 45° with those planes. The cube and the rhombic dodecahedron are thus two special cases of a class of solids formed by rotating square prisms about the axes described. Intermediate angles of rotation give solids which may be new and crystallographically bizarre even if bounded by faces of rational Miller indices. For example, the figure shows a perspective view of intersections of the member of this class bounded by the twelve {210} planes, obtained by rotating the prisms through tan-1(2). The {111} direction is a triad axis, (but the tetrad axes have been destroyed even though the solid has square cross sections). This is just an example of a new way of viewing symmetrical solids of both crystallographic and non-crystallographic importance. The relationships between different members of the same class of solids can best be shown by computer-generated movie films.

20.1-02 ORTHORHOMBIC SPACE-GROUP PROJECTIONS. By Martin Buerger, MIT, Cambridge, MA, U. S. A.

Orthorhombic space groups are ordinarily represented by a "standard projection" although many have up to 6 distinct projections. These can be derived from the "standard projection" thus: Starting with a right-handed axial set a,b,c, the 3 axial planes ab, bc, ca are drawn in the same plane, but with common axes parallel and separated a little. The "standard projection" is mapped on ab, bc, or ca. It is an easy task to draw the symmetry elements in the other two planes. It is helpful to devise a generalized Hermann-Mauguin symbol and write it just above the second axial vector for that plane. For the plane ab the bulk symbol for rotation and screw axes \( a \), and for mirrors and glide planes \( a' \), is \( A \). symmetry similarly related to \( b \) is designated \( B \); symmetry similarly related to \( c \) as \( C \). If the Bravais lattice is designated \( L \) (with primes if the cell is end-centered) the generalized Hermann-Mauguin symbols for projections on ab, bc, and ca are \( L'ABC \), \( L'BAC \), and \( L'CAB \) respectively. These are based on a right-handed system of axes. Three more combinations of the three axial letters are possible by commuting two letters for right-handed axes, thus: \( b'a \); \( c'b \), \( a; b,c \). These generate left-handed projections whose symbols are \( L'ACB \), \( L'CBA \), and \( L'ACB \). All 6 projections have the same origin and are without negative axial vectors. Since they are all generated from the same holding, any one of the 6 can occur on an arbitrary unit-cell face. Not all of the 59 orthorhombic space groups have all projections distinct; 22 have all 6 distinct; 25 have 3 distinct; 1 has 2 distinct, and in 11 all are identical.

20.1-03 ROTATION STEREOHEDRA COMPOUNDS FOR 3 x 23 AND 3 x 432 GROUPS OF SYMMETRY. By V.I. Burdina, Institute of Crystallography, Academy of Sciences of the USSR, Moscow, USSR.

The problem of finding various types of regular partitions upon stereohedra at three-dimensional sphere of rotation S³, as well as at three-dimensional tangential projective space V³ is considered.

For two cases of direct products 3 x 23 and 3 x 432 of the rotation's symmetry groups considered (Burdina V.I. DAN USSR, 246, 1, 96 (1979); and DAN USSR 250, 4, 854 (1980)) the minimal regions of different stereohedra have been reduced to plane regions with a polygonal boundary.

After direct scanning of the regions by computer in steps of 0.025x10⁻⁶, various regions of the same type's stereohedra separated by non-linear boundaries have been recognized. The coordinates of the polyhedra's vertices have been calculated to 0.001 accuracy.

Both the number of faces and the number of vertices for the calculated polyhedra are usually even, but for boundary's stereohedra the number of vertices is always odd.

Based on these partitions, the distributions of coordination 2 compounds may be constructed.

20.1-04 CRYSTALLOGRAPHIC SYMMETRY. By R.Y. Saitulin, Institute of Crystallography, USSR Academy of Sciences, Moscow, USSR.

Geometrical principles of crystallography must be formulated axiomatically for the successful development of crystallographic mathematical methods, but in this case these principles become an independent branch of geometry which may be called "crystallographic geometry". The chief aims of crystallographic geometry are to distinguish rigorously and investigate 1) sets of points corresponding to the centres of atoms in ideal crystal structures, 2) polyhedra corresponding to the crystals. Crystallographic geometry can be formed on the basis of the following system of axioms.

1. Discreteness. The distance between any two points corresponding to the centres of atoms must be greater then some fixed length \( r > 0 \).

2. Coverage. The distance between any point of space and the nearest centre of atom must be shorter then some fixed length \( R > 0 \).

3. Local equality. Each identical atom is in identical environment in the sphere of radius \( 2R \). (It seems very likely that this boundary is equal to \( 4R \). It has been proved for the plane by M.I. Stogrin.)

4. Rationality. There is a coordinate system for each crystal in which every plane containing a crystal face has rational coefficients.