It is shown that the method of generalized representations of the classical space and point groups, and the method of coloured and Q-groups in their magnetic interpretation are the equivalent languages for the symmetry description of magnetic structures arising through the phase transitions in crystals. The colour symmetry methods may be effectively used in the theory of critical phenomena because the scaling transformations of the appropriate Hamiltonians are nothing but a special realization of the colour symmetry conservation law for isolated physical systems. The commensurate (or incommensurate) modulated structure of a crystal arising through the phase transitions may be described in terms of colour space groups isomorphic (or homomorphic) to the initial space groups: \[ \Phi = T \cdot G \cdot T^{-1} \] We use here the non-standard factorization of the space group \( \Phi \) connected with the enlarged unit cell \( \Phi \cdot G \cdot \Phi^{-1} \) and go to the isomorphic positional colour space group \( \Phi \cdot G \cdot \Phi^{-1} \) which is the subgroup of the wreath products of the groups \( \Phi \cdot G \) and \( \Phi \) which correspond to the selected model of imperfect crystal. The action of the generalized symmetry operators \( \Phi \cdot G \cdot \Phi^{-1} \) on the field order parameter \( \eta(G) \) depends on the point coordinates \( (\xi, \eta) \) of \( \Phi \cdot G \cdot \Phi^{-1} \) throughout the volume of \( \Phi \cdot G \cdot \Phi^{-1} \). In the volume of \( \Phi \cdot G \cdot \Phi^{-1} \) one obtains the uniform distribution of \( \eta(G) \). To the scaling transformations \( \Phi \cdot G \cdot \Phi^{-1} \) there correspond the transformations from initial microscopic to some block Hamiltonian. If this result coincides with that of the renormalization group approach and with the experimental data one may take the test model of \( \Phi \cdot G \cdot \Phi^{-1} \) for the representative subgroup of the generalized microscopic symmetry of modulated phase of a crystal in the class of the equivalent symmetry groups. It follows from abstract symmetry conservation law that \( \Phi \cdot G \cdot \Phi^{-1} \) is isomorphic to the group \( \Phi \cdot G \cdot \Phi^{-1} \) and that \( \Phi \cdot G \cdot \Phi^{-1} \) is the common subgroup of \( \Phi \cdot G \) and \( \Phi \cdot G \cdot \Phi^{-1} \) in accordance with the experiment and theory of Landau.
structure factors and the transformed one are identical, i.e. $F(n)=F'(n)$. For the same reason, all transfor-
mations corresponding to a given coset of $G$ in $NE(G)$ result in the same set of indices and related structure
factors. In the present context it is sufficient, there­
fore, to treat one representative symmetry operation
from each coset. Two cases shall be discussed separa-
tely:
(a) The coset can be represented by a pure translation
(1,1). I being the identity matrix: then all indices remain un­
changed (h→h) and only the phases change
$\left(\psi'(n)=-\psi(n)+2\pi b\right)$. The number $n_i$ of such cosets equals
the index between the translation subgroups of $G$ and of
$NE(G)$. The permissible origin translations ($G$.Giacovazzo,
in direct methods may be derived directly as those trans­
lations of $H(G)$ not belonging to $G$ itself.
(b) The coset cannot be represented by a pure trans­
lation: Then each corresponding unit cell transforma­
tion causes a mapping of the reciprocal lattice with the
property, that the two structure factors with the same
indices $F(n)$ and $F'(n)$ (referring to the original basis
and the transformed basis, respectively) are not related
by space-group symmetry, i.e. $F(n)
eq F'(n)$. For this,
$G$ and $NE(G)$ have to belong to different crystal classes.
If $n$ is the index of $G$ in $NE(G)$ and $n_i$ is the index be­
tween the corresponding two translation subgroups, then
$n/n_i$ is the number of symmetrically inequivalent index­
ing schemes which is in the special case of a non-centrosymmetrical crystal structure without
anomalous scatterers Friedel's law holds and, therefore,
the number of inequivalent indexing schemes is reduced
by a factor of 2 (exception: space groups from enantio­
morphic pairs). In such a case different indexing
schemes occur only if $G$ and $NE(G)$ belong to different
lattice groups.

20.2-3 INVARIANT SUBGROUPS OF SPACE GROUPS.
By M. Senechal, Professor of Mathematics,
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Invariant (normal) subgroups play a central
role in group theory, for example in the struc­
ture of groups, in representation theory, and
in group-subgroup relations. We continue our
study of the subgroups of space groups (Acta
Cryst. A 36, 1980, 845-850) by investigating
the properties of invariant subgroups of space
groups, in any dimension. Necessary and suffi­
cient conditions for a subgroup $H$ of a space
group $G$ to be invariant are established: its
translation subgroup $T_H$ must be invariant in $G$,
and the image (factor group) $H/T_H$ must be in­
variant in $G/T_H$. These conditions are re­
stated in a form which leads to a computa­
tional algorithm (tables of invariant sub­
groups are being prepared by F. Engel). The
structures of the images $G/H$ are also dis­
cussed and partially characterized.

20.2-4 GEOMETRIC PROPERTIES OF WYCKOFF SETS IN
SPACE GROUPS.
By Chung Chieh, Guelph-Waterloo Centre for
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A collection of symmetry equivalent points in a space
group is called Wyckoff positions. Most space groups have
positions with point-group symmetry higher than the tri­
ivial, and those with the highest site symmetry are par­
ticularly interesting. For convenience, let those with the
highest site symmetry be called "very special Wyckoff positions". In some space groups, there are several very
special Wyckoff positions that may be permuted by auto­
morphisms of space groups (Koch & Fisher, 1975,Acta Cryst.
A31, 88), and they form a "Wyckoff set", so named by
Wondratschek (International Tables for Crystallography,
sets with the highest site symmetry may also be called
"very special Wyckoff sets".
The Dirichlet domains of the very special Wyckoff
sets of 3-dimensional space groups are polyhedra, which
may be used as geometric units; although their introduc­
tion was for the classification and description of cubic
structure (Chieh, 1979, Acta Cryst., A35, 946), when the concept of geometric unit was employed to clas­
sify tetragonal, hexagonal and rhombohedral space groups
(Chieh, 1983, Acta Cryst., A39, 410), the author has
realized the need for a theoretical basis, i.e. the
rigorous criterion for geometric units. The use of
Dirichlet domains of very special Wyckoff sets seems to be
the most appropriate.

As an example, the geometric units for the 17 2­
dimensional space groups are given in the Figure. There
are four categories reflecting the number of geometric
units per crystallographic cell. Although some may two
types due to the presence (or the lack) of symmetry in the
crystal system. A similar scheme for 3-dimensional space
groups will be presented.

Figure. Geometric units of the 17 2-dimensional space
groups. Site symmetry at the centre of these units
are given in the bracket.