Supplementary Material

Table 1: Group theoretical symmetry-breaking pathways of experimental lattices from the bcc reference lattice. Classified by space group IR and order parameter direction, each pathway shows the point group IR and minimal manifold of SO(3) in Eq. (1) required to achieve it. The order parameter directions are given in an abbreviated form in the notation of (Stokes & Hatch, 2002).

Pathway	S. G. IR	OP Dir.	P. G. IR	$\ell^{\mathrm{req'd}}$
229a → 217a	Γ_2^-	P1	A_{2u}	3
229a → 161a	$H_5^- \oplus \Gamma_2^-$	P3 ⊕ P1	$T_{2u} \oplus A_{2u}$	3
	$H_5^- \oplus \Gamma_4^-$	P3 ⊕ P3	$T_{2u} \oplus T_{1u}$	3
	$H_5^- \oplus H_4^+$	P3 ⊕ P3	$T_{2u} \oplus T_{1g}$	4
	$H_4^+ \oplus \Gamma_2^-$	P3 ⊕ P1	$T_{1g} \oplus A_{2u}$	4
	$H_4^+ \oplus \Gamma_4^-$	P3 ⊕ P3	$T_{1g} \oplus T_{1u}$	4
	$H_5^- \oplus H_2^+$	P3 ⊕ P1	$T_{2u} \oplus A_{2g}$	6
	$H_2^+ \oplus \Gamma_4^-$	P1 ⊕ P3	$A_{2g} \oplus T_{1u}$	6
	$H_1^- \oplus \Gamma_4^-$	P1 ⊕ P3	$A_{1u} \oplus T_{1u}$	9
	$H_1^- \oplus H_4^+$	P1 ⊕ P3	$A_{1u} \oplus T_{1g}$	9
229a → 2i (MEZDIE01)	$N_1^- \oplus \Gamma_4^+$	P1 ⊕ S1	$A_{1u}, E_u, T_{2u} \oplus T_{1g}$	4
	$N_1^- \oplus \Gamma_5^+$	P1 ⊕ S1	$A_{1u}, E_u, T_{2u} \oplus T_{2g}$	4
	$N_2^- \oplus \Gamma_4^+$	P1 ⊕ S1	$A_{2u}, E_u, T_{1u} \oplus T_{1g}$	4
	$N_2^- \oplus \Gamma_5^+$	P1 ⊕ S1	$A_{2u}, E_u, T_{1u} \oplus T_{2g}$	4
	$N_3^- \oplus \Gamma_4^+$	P1 ⊕ S1	$T_{1u}, T_{2u} \oplus T_{1g}$	4
	$N_3^- \oplus \Gamma_5^+$	P1 ⊕ S1	$T_{1u}, T_{2u} \oplus T_{2g}$	4
	$N_4^- \oplus \Gamma_4^+$	P1 ⊕ S1	$T_{1u}, T_{2u} \oplus T_{1g}$	4
	$N_4^- \oplus \Gamma_5^+$	P1 ⊕ S1	$T_{1u}, T_{2u} \oplus T_{2g}$	4
229a → 60c,d (YIMWEW)	*			
229a → 2iii	H_4^-	S1	T_{1u}	3
	H5-	S1	T_{2u}	3

Table 2: Group theoretical symmetry-breaking pathways of experimental lattices for the hcp reference lattice.

Pathway	S. G. IR	OP Dir.	P. G. IR	$\ell^{\mathrm{req'd}}$
194c → 165d	A_2	P3	A_2', A_1'	3
194c → 147d	$\Gamma_3^+ \oplus \Gamma_2^+$	P1⊕ P1	$A_2'' \oplus A_2'$	3
	$\Gamma_4^+ \oplus \Gamma_2^+$	P1⊕ P1	$A_1'' \oplus A_2'$	4
	$\Gamma_4^+ \oplus \Gamma_3^+$	P1⊕ P1	$A_1'' \oplus A_2''$	4
194c → 176h	K_4	P1	E'	3

Notes:

(1) As these tables are not meant to be exhaustive enumerations but only illustrative of the type of potentials necessary to find a given phase transition, we have truncated listings for 161a, 2i (MEZDIE01), and 14e (MECKUA) which have additional pathways similar to those shown.

(2) Inasmuch as a different method was used in this work to choose an embedding of the daughter lattice in the parent lattice than that used in (McClurg and Keith, 2009), the IRs inducing the phase transition from parent to daughter may be different from that work. This, of course, does not affect our numerical results shown in Table 3.

(3) The pathways, space group IRs, order parameter directions, and point group IRs were computed using ISOTROPY. However, transitions belonging to a coupled IR between a high symmetry point and line are currently not a feature of ISOTROPY. These entries have been marked with an asterisk in the table.

Table 3: Group theoretical symmetry-breaking pathways of experimental lattices for the sc reference lattice.

Pathway	S. G. IR	OP Dir.	P. G. IR	$\ell^{\mathrm{req'd}}$
221a → 215a	Γ_2^-	P1	A_{2u}	3
221a → 120c	$R_5^- \oplus \Gamma_2^-$	P1⊕ P1	$T_{2u} \oplus A_{2u}$	3
	$R_4^+ \oplus \Gamma_2^-$	P1⊕ P1	$T_{1g} \oplus A_{2u}$	4
	$R_5^- \oplus R_4^+$	P1⊕ P1	$T_{2u} \oplus T_{1g}$	4
	$R_4^+ \oplus \Gamma_3^-$	P1⊕ P1	$T_{1g} \oplus E_u$	7
	$R_5^- \oplus \Gamma_3^-$	P1⊕ P1	$T_{2u} \oplus E_u$	7