Finite-index normal subgroups of crystallographic space groups

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Symmetry-lowering phase transitions give rise to crystal domain patterns in ferroelectrics and many other types of materials. If the order parameter responsible for the transition possesses child space-group $H$, which is a subgroup of the parent space group $G$, the crystal domains of the child phase are associated with equivalent directions of this order-parameter, which are in one-to-one correspondence with the set $S$ of cosets of $H$ in $G$. While $H$ is not normal in general, the normal core $N$ of $H$ in $G$ is a finite-index normal subgroup of $G$, for which the quotient group $G/N$ has a well-defined permutation action on $S$. In this sense, $G/N$ is the symmetry group of $S$, and can be used to classify its symmetry-inequivalent domain pairs [1-3]. The importance of finite-index normal subgroups to the study of crystal-domain configurations motivates us to tabulate them for each crystallographic space group.