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A First Course in Group Theory. By Bijan Davvaz. Springer, 2021. Softcover, pp. xv + 291. ISBN 978-981-16-6364-2. Price EUR 42.79.

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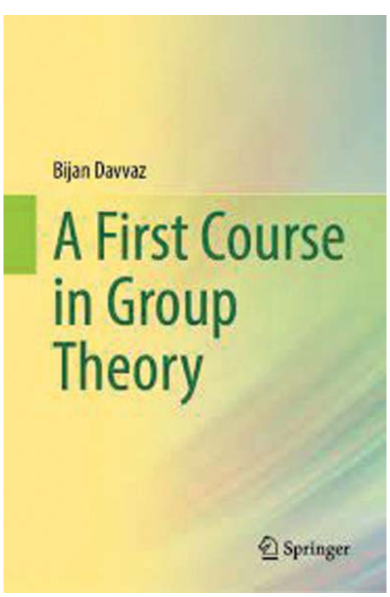

Symmetry is widespread in nature and a common concern of mathematicians and crystallographers. A firm and profound understanding of symmetries requires a reasonable degree of command of group theory, which is part of abstract algebra, a branch of mathematics that deals with abstract algebraic structures such as groups, rings, fields and lattices, among others. The mathematical apparatus of group theory is a means of exploring and exploiting physical and algebraic structures in physical and chemical problems (Butler, 1981). Both theoretical and experimental crystallography cannot abstract from a good knowledge of groups and related concepts. This branch of mathematics has been extensively explored and many well written textbooks are available. However, for a crystallographer, books such as *Topics in Algebra* (Herstein, 2006) and *Algebra* (Artin, 2010) could safely be classified as inaccessible to non-specialists, while books such as *Contemporary Abstract Algebra* (Gallian, 2020) could be accessible but are too voluminous.

A First Course in Group Theory by Bijan Davvaz can serve as a satisfactory introductory textbook in this vein. It is neither an advanced book on group theory nor too voluminous. Each chapter is complemented with several enlightening worked-out examples and eye-catching colourful images. To enhance the further grip of the subject matter, each chapter ends with several engaging and enriching supplementary exercises.

The book consists of 11 chapters that span less than 300 pages. The book can serve as an excellent introductory undergraduate textbook in group theory and, because it is largely self-contained, it can be very useful to crystallographers who lack (but feel an urge for) a basic understanding of the mathematics of symmetries.

Chapter 1 (30 pages), *Preliminary Notions*, lives up to its title. It starts with the fundamental notions that are much needed to understand the literature of groups. The chapter explains the basics of sets and equivalence relations (relations that are reflexive, symmetric and transitive), functions, ordered sets and most importantly combinatorial analysis. Patterns and arrangements are the everyday concern of a crystallographer. The problem of enumerating symmetry patterns that atoms in a crystal may form is of direct concern for a crystallographer, who is required to work in the capacity of a combinatorist (Mala, 2022). The chapter also discusses other basic concepts of number theory such as divisibility and prime numbers.

Chapter 2 (16 pages), *Symmetries of Shapes*, sets the actual tone of the book for a crystallographer. This chapter attempts to explain various symmetries and transformations from very rudimentary levels. From the symmetries of the human form to that of the heart carved out, from basic symmetries of triangles and other geometrical figures to those in the font Geneva, the chapter offers several engaging and easy-to-grasp excursions into the world of symmetries. Using beautiful reflections in nature and particularly those in butterflies and footprints fixed by a glide reflection, the readers are taken on a journey of translations, rotational symmetries, mirror reflection symmetries and congruence transformations.

Chapter 3 (40 pages), *Groups*, starts with a brief history of the subject. It explains binary operations that are pivotal to all groups and pays passing tribute to the giants who contributed to the rich treasure of the literature on this subject. These include the likes of Euler, Lagrange, Abel, Galois and Noether, among others. Some of the content of the chapter such as the discussion of semigroups, monoids and Latin squares has been classified as optional. Sufficient emphasis has been laid on subgroups which are important for any first encounter with group theory.

Chapter 4 (18 pages), *Cyclic Groups*, begins with a discussion of congruences which is a crucial notion concerning the study of the divisibility of integers. Cyclic groups are groups that can be generated by a single element of the group. The modular arithmetic of the clocks gives rise to one of the most basic cyclic groups, the group of integers modulo 12. The notion of cyclic groups has naturally been extended to that of generating sets. All of this is explained via a reasonable exposition of judiciously chosen examples and exercises.

Chapter 5 (26 pages), *Permutation Groups*, discusses the most important groups in mathematics. In the light of Cayley's theorem (discussed in the last chapter) which states that all groups are isomorphic to permutation groups, the chapter brings to the fore the importance and relevance of permutation groups. Permutations are simply bijections from a finite non-empty set to itself. The fact remains that we can construct permutation groups from any finite set of objects. The chapter contains an engaging discussion of the two types of permutations: even and odd. Interestingly, the set of all even permutations gives rise to another important group, the alternating group, while the set of all odd permutations does not do so. The chapter throws sufficient light on the alternating group.

Chapter 6 (18 pages), *Group of Arithmetical Functions*, is more suited for a number-theoretic course than for a group-theoretic one and has, rightly, been classified as optional. It also does not fit into the remit of serving the mathematical needs of a crystallographer.

Chapter 7 (46 pages), *Matrix Groups*, concerns itself with examples of groups that arise out of the study of matrices. Matrices are a daily tool for crystallographers, for representing symmetry operations, switching the coordinate systems and mapping the orientation of a sample, to mention just a few examples. The pick of this chapter is the discussion of rotation groups and the dihedral groups. Dihedral groups come into existence owing to the symmetries of regular or equilateral polygons, such as equilateral triangles and squares. This chapter also discusses the reflections in higher dimensions including those in the plane and the space. A particular exercise asks the readers to try their hands at getting wind of the symmetry groups of certain astonishing snowflakes.

Chapter 8 (22 pages), *Cosets of Subgroups and Lagrange's Theorem*, discusses the special subsets of groups, called the cosets, and the famous Lagrange's theorem that states that the order of a subgroup of a finite group must divide the order of the group. To facilitate an intuitively clear understanding of cosets, the author presents certain revealing geometrical examples of cosets. That is where the book scores more than many other introductory books on the subject. The counting principle for the number of elements in the product of two subgroups and double cosets have also been explained in detail.

Chapter 9 (16 pages), *Normal Subgroups and Factor Groups*, discusses the notions of normal subgroups (groups in which there is no distinction between right cosets and left cosets) and factor groups (groups of all the cosets of a normal subgroup). It also discusses the concept of a class equation. The chapter is quintessential for a good understanding of homomorphism (the subject matter of the last chapter of the book) which relates groups to other groups.

Chapter 10 (14 pages), *Some Special Subgroups*, discusses certain special groups such as the derived group and the maximal subgroup. The chapter can safely be skipped by a first-timer and may not be relevant to a crystallographer.

Chapter 11 (35 pages), *Group Homomorphisms*, is quite an important chapter and makes a fundamental contribution to the understanding of which groups should be considered similar in group-theoretical senses. Engaging discussions on special homomorphisms such as isomorphism and automorphism can also be found. Exploiting these notions, the chapter contains proof of one of the most important and engaging theorems in group theory, the Cayley theorem.

In conclusion, given the amount of literature available on group theory, this book can serve as an excellent introductory textbook on group theory for mathematicians, both amateur and professional, as well as an enriching and engaging textbook for crystallographers. Written by a highly cited group theorist, it is worth the time and money spent on it.

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