Sidorov & Arkhipov (1972) and the calculated curve according to Kachi's model are shown also. The values obtained for the radius of gyration \( R_0 \) and the forward-scattering cross section are presented in Table 1, together with the values calculated from Kachi's model.

Table 1. Radius of gyration \( R_0 \) and forward-scattering cross section \( (\delta r/\delta \Omega)_0 \) derived from experiment

<table>
<thead>
<tr>
<th>Crystal orientation</th>
<th>( R_0 (\text{Å}) )</th>
<th>( (\delta r/\delta \Omega)_0 ) (barn/ster.atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[100]</td>
<td>7.9 ± 0.4</td>
<td>1.80 ± 0.09</td>
</tr>
<tr>
<td>[011]</td>
<td>7.5 ± 0.4</td>
<td>1.59 ± 0.08</td>
</tr>
<tr>
<td>[111]</td>
<td>7.7 ± 0.4</td>
<td>1.55 ± 0.08</td>
</tr>
<tr>
<td>Kachi's model</td>
<td>4.3</td>
<td>2.26</td>
</tr>
</tbody>
</table>

It is likely that the values of Men'shikov et al. are not corrected for slit-height effects. This could explain well the discrepancy. The comparison of our experimental values with the values obtained from Kachi's model shows a remarkable agreement if we take into account the crudeness of the model.

Stimulating discussions with Dr J. Schelten are gratefully acknowledged. Thanks are also due to Mr A. Zinken for his technical assistance.

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References


**Neutron Diffraction by Vortex Lattices in Superconducting Niobium**

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Neutron diffraction experiments on vortex single crystals reveal various information on properties of flux-line lattices. In this paper we present the data obtained from a pure single-crystal niobium rod with its axis parallel to a (111) direction and a Ginzburg-Landau parameter of 0.9. For the largest achievable lattice parameter in this sample of 1940 Å at 4.2 K we found a hexagonal flux-line lattice with an azimuthal mosaic spread of 60° and a radial one of less than 1.2°. From the integrated reflectivity we determined the absolute values of the form factors at the reciprocal lattice points 10, 11, 20, 21, 30 and 31. From these form factors the local magnetic field distribution was calculated.

In the mixed state of type-II superconductors – that is the region between the lower and the upper critical field—the magnetic flux within a sample exists in the form of flux lines (vortices). One can observe these flux lines directly in an electron microscope by the decoration technique which has been developed by Träuble & Essmann (Essmann & Träuble, 1967; Träuble & Essmann, 1968). These electron micrographs have already revealed that flux lines in single-crystal type-II superconductors usually form a hexagonal lattice. The lattice parameter depends on the applied external magnetic field and varies from about 100 Å to several thousand Å. Each flux line carries only one flux quantum of the size of 2.07·10⁻¹⁰ G cm².

The decoration technique yields very good information about the geometrical arrangement of flux lines and especially on lattice irregularities. It does not, however, provide any information about the microscopic magnetic field distribution. This information can only be gained by nuclear methods like the Mössbauer effect (proposed by Sarma, 1964), n.m.r. (Fite & Redfield, 1966; Kung, 1970) or neutron diffraction. The investigation of flux lines by neutrons is an example of two-dimensional crystallography.

Following a suggestion of De Gennes & Matricon (1964), the first neutron scattering experiment on vortices was performed in 1964 (Cribier, Jacrot, Rao & Farnoux, 1964). In the past two years the method of neutron diffraction on vortices has been considerably improved by research groups in Jülich and Saclay (Schelten, Ullmaier & Schmatz, 1971; Cribier, Simon & Thorel, 1972; Schelten, Ullmaier & Lippmann, 1972).
The purpose of this paper is to show what information on properties of flux-line lattices can be gained by means of neutron small-angle diffraction and how the corresponding experiments are carried out. We will restrict ourselves to just one specific sample and will illustrate all of the comprehensive information for one special case where we have the largest possible lattice parameter in the flux-line lattice. This is that value which cannot be exceeded because of the attractive interaction of flux lines in pure niobium even if one lowers the applied field to zero.

In Fig. 1 the reciprocal lattice of a hexagonal vortex lattice is sketched. It is two-dimensional; the extension of the reciprocal-lattice points perpendicular to the lattice plane is inversely proportional to the coherently scattering part of a flux line. If one inserts typical values for the neutron wavelength (\(\lambda = 9 \text{ Å} \)) and the lattice parameter (about 2400 to 600 Å) into the Bragg equation one calculates the range of scattering angles of 15°-60° which is usually experimentally covered.

A neutron diffraction experiment on a type-II superconductor usually yields information on the following flux-line properties:

(i) The scattering angle \(\theta\) allows the calculation of the lattice parameter and the flux density which is the mean value of the microscopic flux density of the flux lines.

(ii) By recording the total diffracted intensity as a function of the rotation angle \(\phi\) of the sample, the symmetry of a flux-line lattice and the azimuthal mosaic spread of a flux-line crystal are determined.

(iii) The radial mosaic spread can be estimated by measuring the intensity as a function of the scattering angle \(\theta\). The integral of the diffracted intensity allows the calculation of form factors of a flux-line lattice from which the microscopic magnetic field distribution of a flux line can be verified. It should be emphasized that neutron small-angle diffraction is the only way to measure form factors of flux-line lattices.

Our results were gained from one specific sample; it was a single-crystal niobium rod with its axis parallel to a \((111)\) direction, a Ginzburg-Landau parameter of 0.9, an upper critical field of 3200 G at 4.2°K and an almost reversible magnetization curve. The data presented all apply to the largest possible lattice parameter in this sample of 1940 Å. In this case we speak of a quasi-isolated flux-line.

For a vortex, single-crystal Bragg scattering only occurs for certain rotation angles \(\phi_h\). The pattern of the intensity distribution (Fig. 2) shows that we have a perfect hexagonal lattice. The distance between the centres of equivalent reflexions differ by \(\pm 15'\) from 60° which is less than the full width at half maximum of the peaks. Each of the six directions of nearest neighbours in the vortex lattice was parallel to one of the \((110)\) crystal directions. The peaks between the intense 10 reflexions are 11 reflexions and are much smaller because of the drastic decrease of the form factor with increasing reciprocal-lattice vector.

We measured the angular distribution of the total diffracted intensity of one of those reflexions with a better angular resolution. Fig. 3 shows rocking curves for a 10 and a 11 reflexion. These rocking curves represent the azimuthal mosaic spread of the vortex crystal. The fact that the shapes and the widths of the two curves are identical within experimental accuracy indicates that the tilting of mosaic blocks causes the major part of the mosaic spread; if the influence of the size of mosaic blocks dominated, the 11 rocking curve...
should be more narrow than that for 10 by a factor of \(\frac{1}{\sqrt{3}}\). We found that the f.w.h.m. of the rocking curves varied with flux density between 30 and 60', where the instrumental resolution was about \(\pm 3\)'.

The diffraction pattern of the intensity vs. the diffraction angle \(\theta\) is shown in Fig. 4. In this measurement a 10 and a T0 reciprocal-lattice point are swept through the Ewald sphere. Our position-sensitive detector records the diffracted intensity on both sides of the primary beam, thus one half of the distance between the centroids of the two peaks is used to determine the scattering angle from which the lattice parameter can be calculated. In all our measurements it agreed with that deduced from the magnetization curve within \(\pm 3\)%. The width of the \(I(\theta)\) vs. \(\theta\) curve is a measure of the radial mosaic spread which is caused by fluctuations of the lattice parameter. The curves in Fig. 4, however, only reproduce our instrumental resolution which is not too good perpendicular to the incident beam and within the scattering plane, although the beam was already collimated to within \(\pm 3\)'. So we can only give an estimation of the radial mosaic spread as better than 1-2' which indicates that the flux-line lattice on average is perfect over a range of at least 15 lattice planes.

The integrated reflectivity is proportional to the square of the form factor which is the normalized Fourier transform of the magnetic field distribution in the unit cell of the flux-line lattice. For the largest achievable lattice parameter we measured the absolute values of the form factor at the reciprocal-lattice points 10, 11, 20, 21, 30 and 31. They are presented in Fig. 5. It should be pointed out that an intensity range of about four decades has been covered.

As has been discussed by Schelten, Ullmaier & Lippmann (1972), we have good evidence that the phases of the form factors for our experimental conditions are all positive. The local magnetic field distribution of this quasi-isolated flux-line arrangement has been calculated by a Fourier inversion as described by Weber, Schelten & Lippmann (1973). For this computation only the first four form factors were taken into account since the higher-order form factors are within the experimental uncertainty of the largest. The truncation error implied by this method is probably small because of the rapid decrease of the higher-order form factors. The result of this calculation is shown in Fig. 6. The maximum reaches about 80% of the upper critical field. The mean flux density is about four times that in the minimum which is only 160 G. One also gets an idea of the shape of a unit cell of a flux-line lattice which contains one flux quantum; the distance between nearest neighbours is 1940 Å.
Fig. 6. Three-dimensional view of the microscopic flux distribution in Nb VI for a flux density of 640 G corresponding to a nearest-neighbour distance between flux lines of 1940 Å. The maximum field at the flux line centre is 2540 ± 90 G = 0.80 Hc2, the minimum field is 160 ± 20 G.

Theoretical considerations of Delrieu (1972) and Brandt (1973) indicate that near Hc2 the form factor F20 becomes negative. In our case this would lead to an interchange of the minimum and saddle-point fields and to a negative field in a region which had been the saddle-point field before. A flux-line regime where field reversal occurs is unstable, however (Kramer & Pesch, 1973). Therefore we doubt whether the theoretical calculations quoted above can be applied to our experimental conditions where the flux density was only 20% of the upper critical field.

References


