Determination of Angles between Blocks from the Topographs Obtained by the Schulz Method

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The paper presents the analysis of formation of the image of non-perfect crystals in the Schulz method. Methods are suggested to determine the angles between blocks by using either 'singular' points or the displacement of characteristic lines. Accuracy of these methods is discussed.

The Schulz (1954) method, which enables X-ray topographs with high angular resolution to be obtained, is widely used to study the degree of perfection of various single crystals. Until now, however, we have had no detailed analysis of how the image of the crystal surface is formed. This impedes the interpretation of the X-ray topographs and leads to errors in calculations of the angles between blocks.*

In this paper the diffraction image obtained by the Schulz method is analysed. The dependence of the image widths of the boundary between blocks on the misorientation angle and on the boundary direction is investigated. The method of determining the angles between blocks is suggested on the basis of the performed analysis.

1. Formation of the image of a perfect segment of the crystal surface

The diagram of the Schulz method is shown in Fig. 1. A point source of polychromatic X-ray radiation is placed at a distance \( D \) from the sample surface. The radiation is diffracted according to the Bragg relation

\[
2d_{hkl} \sin \theta = \lambda,
\]

so that each wavelength \( \lambda \) corresponds to a reflexion along one line on the surface of the crystal. The image on the film is composed of line segments along which the wavelength remains constant. We have derived earlier the equation of such lines for the high-divergence-beam method when the film is placed at a distance \( A \) from the crystal parallel to its surface (Aristov, Shmytko & Shulakov, 1974a):

\[
R(\theta, \varphi) = \frac{\cos \theta}{\cos \alpha} \left( \frac{D}{\sin (\theta - \theta_1)} + \frac{A}{\sin (\theta + \theta_1)} \right) \quad (1)
\]

Here \( \alpha \) is the angle between the reflecting planes and the crystal surface; \( \theta_1 = \arcsin (\sin \alpha \cos \varphi) \); \( R(\theta, \varphi) \) and \( \varphi \) are polar coordinates of the diffraction lines on the film; the origin \( O \) of the coordinate system is located at the intersection point of the film plane with the normal to the reflecting planes projected from the radiation source \( S \); the angle \( \varphi \) is measured from the direction of the projection of the normal into the film. The crystal in the Schulz method is oriented in such a way that reflexions can be obtained from the planes whose zone axis lies in the azimuthal direction.†

Usually the crystal dimensions are small as compared to the distance \( R(\theta, \varphi)A=0 \). Taking into account the crystal orientation and neglecting the second-order terms in equation (1) we can assume the value \( R(\theta, \varphi) \) to be independent of the angle \( \varphi (\varphi \ll 1 \text{ and } \theta_1 = \alpha) \), and the segments of lines of constant \( \lambda \) to be segments of circles for any value of the angle \( \alpha \).

Let us establish within the assumptions made above the relationship of the elements of the crystal surface to their images on the film. Let us define the magnification coefficients \( P_r \) and \( P_a \) in the radial and azimuthal directions as the ratios of respective lengths of segments on the image to their lengths on the crystal. Using equation (1) we obtain:

\[
P_r = \frac{\Delta R}{\Delta R_{A=0}} = 1 + \frac{A \sin^2 (\theta - \alpha)}{D \sin^2 (\theta + \alpha)} \quad (2.1)
\]

\[
P_a = \frac{\Delta \varphi R}{\Delta \varphi_{R=0}} = 1 + \frac{A \sin (\theta - \alpha)}{D \sin (\theta + \alpha)} \quad (2.2)
\]

Here \( \Delta R = R_2 - R_1 \), \( \Delta \varphi = \varphi_2 - \varphi_1 \). At \( \alpha \neq 0 \) the magnification coefficients \( P_r \) and \( P_a \) are not identical and change in the radial direction as the angle \( \theta \) changes. This results in a distortion of shapes of crystal surface elements on the topograph. For instance, the direction in the crystal making the angle \( \gamma \) with \( e_\alpha \) corresponds to the direction on the image making with \( e_\alpha \) the angle \( \gamma' \):

\[
\tan \gamma' = \frac{P_r}{P_a} \tan \gamma \quad (3)
\]

Thus we have established that the image of the crystal surface is formed by segments of circles of

* Several methods of calculating misorientation angles from the Schulz X-ray topographs are known (Umanskii, 1967; Meleshko & Sosnina, 1971; Kostyukova, 1973). We shall demonstrate below that application of these methods yields erroneous results.

† We shall call the radial direction on the film \( e_\alpha \) the direction lying in the plane comprising the incident and reflected beams [this direction coincides with that of the radius \( R(\theta, \varphi) \) drawn from the origin of coordinates \( O \)], and call the azimuthal direction \( e_\varphi \) that normal to this plane (see Fig. 1).
constant \( \lambda \). At \( \alpha \neq 0 \) the shape of surface elements is distorted, angles between the directions in the crystal are not invariant on the image and both the angle \( \theta \) and the magnification coefficients change along the radial direction. On the other hand, \( \gamma' = \gamma \) and the image is free of distortions at \( \alpha = 0 \).

2. Images of boundaries between blocks

Let us consider two blocks on the crystal surface separated by the small-angle boundary. Misorientation of one block in relation to the other can be presented as a vector \( \mathbf{L} \) with the length equal to the misorientation angle \( \varepsilon \) while its direction coincides with the block rotation axis. At \( \varepsilon \ll 1 \) the vector \( \mathbf{L} \) can be presented as the sum of rotations about three mutually normal axes

\[
\mathbf{L} = \delta_r \mathbf{i} + \delta_a \mathbf{j} + \delta_n \mathbf{n},
\]

where \( \mathbf{i} \) is parallel to the azimuthal direction \( \mathbf{e}_\alpha \), \( \mathbf{n} \) is the normal to the reflecting planes, \( \mathbf{j} \) lies in the incidence plane and is parallel to the reflecting planes. We shall call \( \delta_r \) the radial, \( \delta_a \) the azimuthal and \( \delta_n \) the normal components of the projection of the vector \( \mathbf{L} \). Misorientation of reflecting planes, which is determined by the components \( \delta_r \) and \( \delta_{\phi} \), causes shifting of block images. The boundaries between blocks are imaged on the topograph as light and dark bands corresponding to overlapping or divergence of the beams reflected by the neighbouring blocks. The analysis of formation of images of boundaries between blocks will be carried out by means of equation (1).

The position of the origin of the chosen coordinate system \((R, \varphi)\) depends on the orientation of the reflecting planes in relation to the crystal surface. In the case in question, the origins of coordinates in the first and second blocks will be shifted in the crystal in relation to one another by \( D \delta_r / \cos^2 \alpha \) in the radial direction, and by \(-D \delta_{\phi} / \cos \alpha \) in the azimuthal direction. These shifts on the film are equal to \((D - A) \delta_r / \cos^2 \alpha \) and \((A - D) \delta_{\phi} / \cos \alpha \). We shall neglect the width of the boundary between blocks and assume that each point of the boundary belongs to both blocks. Let us consider a point on the boundary separating two blocks. Owing to misorientation of blocks in the radial direction, the X-rays are reflected at this point at different Bragg angles \( \theta + \delta_{\phi} \) and \( \theta \), and, owing to the azimuthal misorientation, the angular coordinate \( \varphi \) of the boundary point is different for the two blocks, so that \( \varphi' = -D \delta_{\phi} / \cos \theta R_{\alpha} = 0 \). Two images of the boundary point will appear on the film, the distance between these images being determined by the misorientation angle of the reflecting planes. We denote the distance between these images in the radial direction as \( b_r \) and that in the azimuthal direction as \( b_a \). Taking into account the foregoing considerations we calculate these distances to the accuracy of the second-order terms:

\[
b_r = R(\theta + \delta_{\phi} - \delta_r) - R(\theta, \alpha) + \frac{D - A}{\cos^2 \alpha} \frac{2A \delta_r}{\sin^2(\theta + \alpha)} \quad (5.1)
\]

\[
b_a = -R(\theta, \alpha) A \delta_{\phi} + \frac{A - D}{\cos \alpha} = \frac{2A \sin \delta_{\phi}}{\sin (\theta + \alpha)}. \quad (5.2)
\]

The situation described above is shown in Fig. 2. The point \( B \) on the block boundary is represented by points \( B_1 \) and \( B_2 \) in the images of the first and second blocks respectively. We can see that the width of the straight-line segment of the boundary depends both on the values of \( b_r \) and \( b_a \) and on the boundary direction:

\[
d = b_r \cos \gamma' - b_a \sin \gamma'. \quad (6.1)
\]

Using equations (5) and expressing \( \delta_r \) and \( \delta_{\phi} \) via the misorientation angle of the reflecting planes \( \delta(\delta_r = \delta \cos \xi, \delta_{\phi} = \delta \sin \xi) \) we obtain

\[
d = -\frac{2A \delta}{\sin^2(\theta + \alpha)} \left[ \cos \xi \cos \gamma' \right. \\
+ \sin \xi \sin \gamma' \sin \theta \sin (\theta + \alpha) \left]. \quad (6.2)
\]

Here \( \xi \) is the angle between the azimuthal direction \( \mathbf{e}_\alpha \) and the projection of misorientation axis \( \mathbf{L} \) on the reflecting planes, \( \delta \) is the absolute value of the angle; \( d < 0 \) means that block images are superposed and the boundary is observed on the topograph as a dark band. As follows from Fig. 2 and equations (6), the width of the boundary image at the same value of \( \delta \) can vary as function of its direction, including zero width; the boundary may appear both as a light and as a dark band. We can see from equation (6.2) that the width of the boundary image is not constant since the Bragg angle may change along the boundary direction. This variation is described by the second-order terms.

To check the foregoing arguments we have run the following experiment. A series of X-ray topographs
were obtained on the NaCl crystal with the reflecting planes (100) parallel to the cleavage surface (\( \alpha = 0 \)), \( \theta \sim 18^\circ \) and with the distances \( A \) and \( D \) equal to 45 mm and 60 mm respectively. The experiment was conducted on a 'Microflex' unit with the silver anode. The focal spot diameter did not exceed 20 \( \mu \text{m} \). Between exposures the crystal was rotated around the normal to the crystal surface so that the angle \( \gamma \) was changed. Fig. 3 shows as examples three topographs of the same area of the crystal surface. These topographs were obtained for various values of the angle \( \gamma \) which were selected in such a way that the boundary between two large blocks 1 and 2 appeared zero-wide [Fig. 3(b)], light [Fig. 3(a)] and dark [Fig. 3(c)].* In block 2 we also observe one more boundary, dark in Fig. 3(a) and turning light in Figs. 3(b) and 3(c). The width of the boundary between blocks 1 and 2 was measured by means of these topographs. Fig. 4 shows the dependence \( d(\gamma) \) plotted according to equation (6.2) at \( \alpha = 0 \). In this case \( \zeta = \gamma + \eta \), where \( \eta \) is the angle between the boundary direction and the projection of the misorientation axis \( L \) on the reflecting planes. The values of the angles \( \delta \) and \( \eta \) were found by the method presented below. The plot in Fig. 4 also contains the experimental points which are seen to agree well with the calculated curve. This means that the approximation of equation (6.2) provides a good description of the experimental situation.

We have thus demonstrated that the width of the boundary between blocks can be found with sufficient accuracy from equation (6.2). Neither \( \delta_r \) nor \( \delta_a \) nor \( \delta \) can be found from the boundary image width since the angle \( \xi \), i.e. the orientation of the rotation axis in relation to the crystal, is not known. Only when the boundary is parallel to either the azimuthal or the radial direction one can determine the angle component \( \delta \) parallel to the boundary direction, from the boundary width.

We now analyze the method of determination of angles between blocks from the Schulz X-ray topographs at an arbitrary orientation of the boundary.

3. Determination of angles between blocks

3.1 Analysis of known methods

We have mentioned in the abstract that formerly suggested methods of determining the misorientation angles in the Schulz method are not correct and yield varying values of the angle between the same blocks. These methods are sufficiently well known and have been used a number of times in various investigations. This fact forces us to dwell on the analysis of errors in these methods.

(a) It was assumed by Umanskii (1967), Meleshko & Sosnina (1971) and Kostyukova (1973) that the widths of the boundary band between block images in the azimuthal (\( m_a \)) and radial (\( m_r \)) directions for a given geometrical arrangement depend only on \( \delta_r \) and \( \delta_a \), respectively. It follows from equation (6.2) that since both \( m_a \) and \( m_r \) are proportional to \( d \), both of these values (\( m_a \) and \( m_r \)) are functions of both \( \delta_r \) and \( \delta_a \) simultaneously and cannot be used to determine the angles between blocks. The photograph in Fig. 3(b), for example, corresponds to the case in which \( d \), \( m_a \) and \( m_r \) are equal to zero at non-zero \( \delta_a \) and \( \delta_r \).

(b) It was suggested by Umanskii and Kostyukova that \( \delta_a \) be measured from the discontinuity of the characteristic line at the block boundary by the use of equation \( 2A\delta_a = c_a \) where \( c_a \) is the azimuthal displacement of the characteristic line ends in the gap (the case when \( \alpha = 0 \)). We have shown earlier that determination of the value of \( \delta_a \) by this formula leads to considerable errors since \( c_a \) depends not only on \( \delta_a \) but also on \( \delta_r \) and on the angle \( \gamma \) (Aristov et al., 1974a). This last fact was reported by Yalzev & Rusakov (1967), who suggested that \( \delta_a \) be calculated by means of a somewhat different formula: \( 2A\delta_a = c_a + c_r \cot \gamma \) where \( c_r \) is the displacement of the characteristic line ends at the block boundary in the radial direction. However, the derivation of this formula did not take into account the focusing of characteristic X-rays reflected by different blocks in the radial direction. The correct formula will be given below [equation (11)].

Thus the application of none of the published methods enables us to derive the true misorientation of blocks.

3.2 The method of determination of block misorientation (a). Determination of the angle by means of singular points

As follows from the foregoing analysis, equations (5) can be used if the distances \( b_r \) and \( b_a \) between the images of one and the same point of the block boundary

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* The photographs in Fig. 3 have reversed contrast so that superposition of block images appears as a light band while the gap between two images forms, on the contrary, a dark band.

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Fig. 2. Formation of the block boundary image. Points \( B_1 \) and \( B_2 \) correspond to one point of the crystal; \( b_r \) and \( b_a \) are displacements of these points in the radial and azimuthal directions, respectively; \( d \) is the block boundary width on the topograph, \( \gamma' \) is the angle between the azimuthal direction and the boundary direction on the topograph.
are measured. An acute angle, a rupture, a scratch intersecting the boundary, etc. can be used as such a point (see Figs. 2, 3). In case such 'singular' points are difficult to find on the image of the block boundary fine wires can be placed between the crystal and the source. Their images will be jogged at the block boundaries, the ends of the gap corresponding to one point in the crystal. The displacement of the wire-image ends in the gap will also be determined by equation (5) and will be independent of the wire position relative to the crystal. It is advisable to place the wires close to the crystal surface since in this case the image will be sharpest. Fig. 5 presents a topograph of a model silicon crystal, consisting of two blocks [see § 3.3(5)]. Copper wires 50 µm in diameter were placed before the crystal at a distance of 0.5 cm. Distinct shadows of the wires are seen in the photograph. Gap measurement was effected with an accuracy of 20 µm.

The photographs (Fig. 5) exhibit the characteristic Ag Kα line. From this we can readily determine the radial and azimuthal directions and use this line as a reference to find the Bragg angle 0. The gap in the characteristic line [Fig. 5(b)] can also be used to determine the misorientation between blocks.

(b). Angle determination from the gap in the characteristic line

We derive from equation (1) that the displacement of characteristic lines in the radial direction \( c_r \) is determined by the radial misorientation of blocks:

\[
c_r = \left( \frac{D}{\sin^2 (\theta - \alpha)} - \frac{A}{\sin^2 (\theta + \alpha)} \right) \delta_r .
\]

For the azimuthal direction, we must take into account, not only \( \delta_r \), but also that the beams, corresponding to the gap in the characteristic line, are reflected at different points of the crystal. We find the displacement of characteristic lines in the azimuthal direction from equations (1-3):

\[
c_a = 2A \frac{\sin \theta \delta_a}{\sin (\theta + \alpha)} + P_a c^*_a .
\]

where \( c^*_a(c^*_r) \) is equal \( c_a(c_r) \) at \( A = 0 \).

If the boundary is rectilinear, then \( c^*_a \) for neighboring blocks is found from the expression

\[
c^*_a = c^*_r \cot \gamma = \frac{D \delta_c \cot \gamma' P_r}{\sin^2 (\theta - \alpha) P_a} .
\]

If the angle \( \gamma' \) cannot be measured on the topograph we have to perform two exposures at different distances \( A \). The unknown value \( c^*_a \) can then be eliminated from equation (11.1) by measurement of the azimuthal displacements of the line for the first and second exposures. Thus the block misorientation can be derived from the Schultz X-ray topographs by using singular points on the boundary, discontinuities of wire shadows, or gaps in characteristic lines. We shall now proceed to the analysis of accuracy obtained with these methods.

3.3 The accuracy of determining the block misorientation angles

1. Dimensions of the focal spot affect both the angular distribution and the accuracy of determination of the misorientation angle. This is because the image of each point of the crystal surface spreads on the topograph to the size

\[
fA \sin (\theta - \alpha)/D \sin^2 (\theta + \alpha)
\]

where \( f \) is the dimension of the focal spot projection onto the plane normal to the direction of the incident X-ray beam. Equations (5) show that only those blocks that are misoriented in the azimuthal and radial directions by angles exceeding \( \delta_r(\min) \) and \( \delta_a(\min) \),

\[
\delta_r(\min) = \frac{f \sin (\theta - \alpha)}{2D \sin \theta}
\]

\[
\delta_a(\min) = \frac{f \sin (\theta - \alpha) \sin (\theta + \alpha)}{2D}
\]

will be observed in the topograph.†

† Angular resolution in the characteristic spectrum at distances \( D \) smaller than \( f \sin (\theta - \alpha)/\Delta \theta \) will be determined in the radial direction only by the spectral width of the characteristic line \( \Delta \theta \).

† Here we assume that resolution of the photographic film is better than that of the method [see equation (12)].

![Fig. 3](image-url) An example of the topograph of a segment of the NaCl crystal obtained at different positions of the crystal. Contrast on the photographs is reversed in relation to that of the topographs. (a) corresponds to the superposition of block images \( d < 0 \). (b) The boundary image width is zero \( d = 0 \). (c) Block images are separated \( d > 0 \).
From equation (13) we obtain that the angular resolution of the method is higher in the radial direction than in the azimuthal direction, and that it improves as the distance D increases. The problem of selecting the optimal exposure conditions is analyzed in more detail by Kostyukova (1973). We should note that the accuracy of obtaining misorientation angles can be increased in some cases by photodensitometry of line ends in the gap (Alaverdova, Panchekha & Fuks, 1972).

2. According to equations (5), (10) and (11), the error in determining block misorientation depends mainly on the accuracy of measuring coordinates of points on the photographic film. The best results are obtained on well annealed crystals with large blocks. It should be emphasized that oblique incidence of the beam onto the film allows the use of only thin-layer emulsions.

3. The accuracy of methods (a) and (b) can be improved when two topographs obtained with different distances, \( A_1 \) and \( A_2 \), are used. This is because the distance \( \Delta Z = A_2 - A_1 \) can be measured to a higher accuracy than either \( A \) or \( D \). In this case calculations are performed by means of equations (5.1), (5.2), (10), (11.1) and (11.2) transformed into the difference form:

\[
\begin{align*}
 b_{r2} - b_{r1} &= \frac{2\Delta Z}{\sin^2(\theta + \alpha)} \delta_r, \\
 b_{a2} - b_{a1} &= \frac{2\Delta Z \sin \theta}{\sin (\theta + \alpha)} \delta_a, \\
 c_{r2} - c_{r1} &= \frac{\Delta Z}{\sin^2(\theta + \alpha)} \delta_r, \\
 c_{a2} - c_{a1} &= \frac{2\Delta Z \sin \theta}{\sin (\theta + \alpha)} \delta_a + \frac{\Delta Z \cot \gamma'}{\sin (\theta + \alpha) \sin (\theta - \alpha)} \delta_r,
\end{align*}
\]

where \( b_{r2}, b_{a2}, c_{r2} \) and \( c_{a2} \) were measured on a topograph obtained at a distance \( A_2 \), and \( b_{r1}, b_{a1}, c_{r1} \) and \( c_{a1} \) on one obtained at a distance \( A_1 \).

4. All formulae for calculation of block misorientation are obtained in the linear approximation. Taking the second-order terms into account meets with difficulties because this requires more precise knowledge of the geometry of the setup as well as high accuracy in measurement of the orientation of the reflecting planes. We shall therefore limit the discussion to the analysis of the accuracy of the linear approximation.

First we must emphasize that in calculations of the radial displacement of the characteristic lines most of the second-order terms increase proportionally to the increasing distance \( A + D \) while the linear term is proportional to \( (A - D) \) (here we assume for the sake of simplicity \( \alpha = 0 \)). This means that at \( A \sim D \) the value of \( \delta_r \) cannot be found from the displacement of the...
characteristic lines. The accuracy reaches maximum at $A \ll D$ and $A \gg D$. Determination of $\delta_i$ by the double-exposure method provides the accuracy for the cases $A \ll D$ and $A \gg D$ since the first- and second-order terms are proportional to $A_2 - A_1$.

It must be emphasized that the accuracy of the foregoing methods is limited by the accuracy to which we know the orientation of the reflecting planes of the crystal. An error of about $1^\circ$ at $\theta \approx 20^\circ$ results in an accuracy of calculating the misorientation angle not better than 5%. As the reflection angle increases, all errors decrease approximately proportionally to $\text{c tg} \theta$.

5. It has already been mentioned above that verification of the aforementioned techniques and evaluation of the accuracy of determining the misorientation angle were carried out on a model silicon single crystal that consisted of two blocks. A single crystal was cut and then glued together in such a manner as to produce a model of a crystal consisting of two large blocks separated by a small-angle boundary. The boundary width did not exceed 5 $\mu$m. The use of this 'model' crystal made it possible to estimate the errors inherent in the above methods and to compare them. Table 1 lists the results of measuring the components of the rotation angle between two blocks. X-ray topographs were obtained at different distances $A$ ($D = 54$ mm; $\theta \approx 17^\circ$); the reflecting planes $(110)$ coincided with the crystal surface to the accuracy of $1^\circ$. The results given in the last three columns correspond to the double-exposure method [equations (14.1)-(14.4)]. This table shows that the accuracy is higher at larger distances $A$ and when the angle is determined by the double-exposure techniques. The value of $\delta_i$ can be determined from the displacement of characteristic lines on Schultz topographs only by the double-exposure techniques. Similar series of experiments were run at different positions of the samples. The angles $\delta$ calculated in different series coincided to the accuracy of several percent.

Thus the theoretical analysis of errors of the methods and the control tests indicate that misorientation of blocks can be measured on the Schultz topographs with satisfactory accuracy.

3.4. **Determination of the total angle of block misorientation.**

X-ray topographic methods allow the determination in a straightforward manner of the misorientation of the reflecting planes in the blocks. To find the total angle between blocks indirect methods are applied. These methods are based on the determination of misorientation of several sets of planes in the blocks. A general discussion of this problem can be found in Aristov et al. (1974b).

The total angle of misorientation of two blocks is conveniently found in the Schulz method from two topographs obtained from two different sets of reflecting planes, $(h_1k_1l_1)$ and $(h_2k_2l_2)$ (it should be remembered that the zone axes of these planes are parallel to the azimuthal direction). The components $\delta_{1\parallel}$ and $\delta_{1\perp}$, corresponding to the misorientation of the reflecting planes $(h_1k_1l_1)$, are obtained from the first topograph. The second topograph yields only the azimuthal misorientation of the reflecting planes $(h_2k_2l_2)$ (we shall denote it by $\delta_{2\parallel}$). Using the relation $\delta_{2\parallel} = (L_2 L_1)$ and expanding the vector $L$ in the coordinate system $(i_1, j_1, n_1)$ related to the reflecting planes $h_2k_2l_2$ [see equation (4)], we obtain

$$\delta_{2\parallel} = \delta_{1\parallel} \cos \beta + \delta_{1\perp} \sin \beta$$

(15)

where $\beta$ is the angle between the reflecting planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$. The unknown component $\delta_{2\perp}$ can be found from equation (15). The components $\delta_{1\parallel}$, $\delta_{1\perp}$, $\delta_{2\parallel}$ completely define the vector $L$ and, consequently, the misorientation between blocks.

**Conclusions**

We have presented a thorough analysis of the Schultz method. It is shown that the image of the crystal surface on the topograph can yield the shape [see equation (3)], size [equation (2)] and misorientation [equations (5), (10), (11), (14) and (15)] of blocks. All equations [with the exception of equation (15)] are presented for the case of recording the topograph on a film parallel to the crystal surface; however, the results obtained are readily generalized to the case when the film is placed normally to the diffracted beam. It should be taken into account that the magnification coefficient in the radial direction is changed, so that we have to replace $y'$ by $y',_1$, where $y',_1 = \arctg [\text{tg} y' \sin (\theta + \alpha)]$; $b_\perp$, $c_\perp$ by $b_\perp/\sin (\theta + \alpha)$, $c_\perp/\sin (\theta + \alpha)$; $A$ by $A_\perp \sin (\theta + \alpha)$ where $A_\perp$ is the film-to-crystal distance along the reflected beam.

The paper suggests a method of determining the angle between blocks from the Schulz X-ray topographs. Other topographic methods, the Fujiwara and Berg-Barrett methods, differ from the Schulz method in the geometrical arrangement, so that the formulae relating the linear and angular dimensions must be different. However, the conclusion that the boundary

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<th>Table 1. Misorientation angle between blocks</th>
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<td>Method</td>
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<tr>
<td>$\delta_i (\times 10^{-2}$ rad)</td>
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<td>$\delta_i (\times 10^{-1}$ rad)</td>
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width does not give the measure of the misorientation angle also remains valid for these methods. The boundary width has previously been used to extract the misorientation angles both in the Schulz method and in other topographic methods. The results obtained when these methods were used require thorough reconsideration on the basis of the present paper.

References


