Projective Properties of Laue Topographs

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Abstract
White beam or transmission Laue X-ray topography is a now widely used imaging technique, especially in association with synchrotron sources. Several images may be recorded simultaneously, potentially allowing for direct defect characterization. Owing, however, to projection conditions, these images suffer from rather large geometrical distortions. This paper presents a general treatment of the projective properties of Laue topographs. Specific results applicable to a simple but frequently utilized specimen-detector geometry are also given. An example serves to demonstrate the usefulness of such calculations in the interpretation of experimental results.

Introduction
This paper presents a simple projective geometry calculation applicable to white-beam X-ray topography such as widely used in conjunction with synchrotron sources. White-beam topography is basically a transmission Laue experiment (Guinier & Tennevin, 1949; Tuomi, Naukkarinen & Rabe, 1974) in which defects or other lattice imperfections may be seen inside the Laue spots due to the small divergence of the beam.

Since usual detectors are photographic plates, which, geometrically, are planes, severe image distortions are observed in the various spots. It is important to know these distortions and more generally to know the projection properties of such an experiment if the ability of the Laue technique of producing several reflections simultaneously is to be fully exploited.

This calculation aims to:
(i) compute the position of the spots on the detector and the operative wavelengths;
(ii) define the trace on the detector plane of the plane of incidence of a given reflection;
(iii) find the trace on the detector of a given crystallographic direction in the sample for a given spot, and the magnification ratio along that direction;
(iv) define the trace on the detector of a given crystallographic plane for a given spot as well as the projected width of that plane.

I. Reference axes
The reference axes systems are shown in Fig. 1.

The $X_i'$ system is linked to the incident beam, $X_1'$ being the direction of the incident beam $s_0$.

The $X_i$ system is linked to the detector plane, $X_1$ being perpendicular to that plane.

The $X_i$ set of axes has its origin in the detector plane the equation of which is simply

$$X_1 = 0 \quad (1)$$

which may also be expressed as

$$X_1 = d. \quad (1')$$

Finally, the $X_i'$ set of axes is linked to the sample, $X_3'$ belonging to the sample surface and to the $X_1, X_3$ plane, $X_1'$ being normal to the sample surface, $X_2 = X_3 \times X_1$.

The first step in the calculation consists in transforming any crystallographic direction in the sample into the $X'$ set of axes: a unit vector $v$ expressed in a set of orthogonal axes $i_1, i_2, i_3$ linked to the crystal becomes

$$(V') = (A)(v) \quad (2)$$

* The $i_1, i_2, i_3$ set of axes can obviously be the crystal axes for cubic crystals. For non-cubic crystals, a proper transformation of the crystal axes into a convenient set of orthogonal axes must first be performed.

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where the matrix \((A)\), which is assumed to be orthogonal, is defined by
\[
(X') = (A)(i). \tag{3}
\]

In the following, any quantity appearing with a single (double) prime is assumed to refer to the \(X'(X')\) set of coordinates. When no prime appears it refers to the \(X\) set of axes. Small letters are used when a quantity is referred to the \(i\) set of axes.

Other axes transformations read:
\[
(X') = (B)(X'') \tag{4}
\]
\[
(X') = (C)(X'') \tag{5}
\]
\[
(C) = \begin{pmatrix}
\cos 2\gamma & 0 & \sin 2\gamma \\
0 & 1 & 0 \\
-\sin 2\gamma & 0 & \cos 2\gamma
\end{pmatrix}. \tag{6}
\]

\(\gamma\) is defined in Fig. 1. It is assumed to be positive according to the usual anticlockwise trigonometric convention.

Finally, one has
\[
\xi_1 = d - X_1 \\
\xi_2 = -X_2 \\
\xi_3 = X_3 + d \tan 2\gamma, \tag{7}
\]
where \(d\) is defined in Fig. 1.

An important practical geometry is met when the detector is perpendicular to the exit beam of a symmetric Laue reflection (Fig. 2). If this is so, distortion is minimized for one of the spots, which we shall hereafter call the main spot. Therefore, in addition to general equations, specific results pertaining to this geometry are given in the Appendix.

### II. Position of the spots and operative wavelength

The reflection is assumed to occur off a plane with normal \([hkl]\) expressed in the crystal axes. Let us call \(n\) a unit vector parallel to the plane normal.

The plane of incidence (Fig. 3) contains, by definition, the incident beam direction \(s_0\), the reflecting-plane normal \(n\) as well as the reflected direction \(s_h\). \(s_0\) and \(s_h\) are taken as unit vectors.

The position of each spot is determined by the intersection of a straight line parallel to \(s_h\) and passing through the origin \(O\) with the detector plane. Since the \(s_h\) direction belongs to the \(s_0, n\) plane, one has
\[
s_h \cdot s_0 \times n = 0. \tag{8}
\]

The angle between \(s_h\) and \(s_0\) is \(2\theta_{hkl}\), \(\theta_{hkl}\) being the Bragg angle for the \(hkl\) reflection. Therefore
\[
s_h \cdot s_0 = \cos 2\theta_{hkl} \tag{9}
\]
provided \(|s_h| = 1\) and
\[
\theta_{hkl} = |\arcsin (s_0 \cdot n)|. \tag{10}
\]

The operative wavelength is deduced from Bragg's law
\[
\lambda_{hkl} = 2d_{hkl}|s_0 \cdot n| \tag{11}
\]
where \(d_{hkl}\) is the interplanar spacing.

The calculation of the components of \(s_h\) is most easily performed in the \(X''\) set of axes. One gets
\[
S_h' = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{12}
\]
\[
(N') = (\hat{B})(A)(n) = (\hat{B})(N') \tag{13}
\]
\[
S_h = \begin{pmatrix}
\cos 2\theta_{hkl} \\
\sin 2\theta_{hkl}N_2/(N_2^2 + N_3^2)^{1/2} \\
\sin 2\theta_{hkl}N_3/(N_2^2 + N_3^2)^{1/2}
\end{pmatrix}. \tag{14}
\]

The equation of a straight line passing through \(O\) and parallel to \(S_h\) is simply given by
\[
\frac{X_1}{S_{h1}} = \frac{X_2}{S_{h2}} = \frac{X_3}{S_{h3}} \tag{15}
\]
where
\[
(S_h) = (C)(S_h) \tag{16}
\]
or, according to (7),
\[
\frac{d - \xi_1}{S_{h1}} = \frac{-\xi_2}{S_{h2}} = \frac{\xi_3 - d \tan 2\gamma}{S_{h3}} \tag{17}
\]
where \(d\) is the sample-to-detector distance (see Fig. 1). It intercepts the detector plane \(\xi_1 = 0\) at the point defined by
\[
\xi_2 = -d(S_{h2}/S_{h1}) \\
\xi_3 = d \left[\tan 2\gamma + (S_{h3}/S_{h1})\right]. \tag{18}
\]

### III. Trace of the plane of incidence

The plane of incidence for the \(hkl\) reflection contains \(s_0\) and therefore contains the origin \(O'\) in the detector plane and the point on the detector defined by (18).
The angle made by the trace, in the detector plane, of the plane of incidence with the $\xi_2$ axis (Fig. 4) is therefore
\[ \varphi = \arctan \left( \frac{\xi_3}{\xi_2} \right). \] (19)

IV. Projection of a crystallographic direction \([uvw]\) and magnification ratio

Let us call \(\mathbf{u}\) a unit vector parallel to the \([uvw]\) direction, expressed in the crystal axes.

The projection of the \([uvw]\) direction is determined by the intersection of the detector plane with the plane containing \(\mathbf{u}\) and \(\mathbf{s}\). This plane is defined by
\[ \begin{vmatrix} X_1 & X_2 & X_3 \\ S_{h1} & S_{h2} & S_{h3} \\ U_1 & U_2 & U_3 \end{vmatrix} = \text{constant}, \] (20)

where \(S_h\) is given by (16).

\[ (\mathbf{U}) = (C)(B)(\mathbf{U}'); \quad (\mathbf{U}') = (A)(\mathbf{u}). \] (21)

The intersection of \((\mathbf{U})\) with the detector plane satisfies (20) and \((1'\)) of the projection of the \([uvw]\) direction and the $\xi_2$ axis (Fig. 4) is found to be
\[ \psi = \arctan \left( \frac{S_{h3}U_1 - S_{h1}U_3}{S_{h1}U_2 - S_{h2}U_1} \right). \] (22)

Let us now consider the unit vector \(\mathbf{u}\) which has its origin at \(O\). The coordinates of the extremity \(H\) of this vector are
\[ H = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}. \] (23)

The extremity of this vector projects along \(S_h\) onto the detector plane. The straight line parallel to \(S_h\) passing through that point has the following equation:
\[ \frac{X_1 - U_1}{S_{h1}} = \frac{X_2 - U_2}{S_{h2}} = \frac{X_3 - U_3}{S_{h3}}. \] (24)

It intercepts the \(X_1 = 0\) plane, i.e. a plane parallel to the detector and passing through the origin \(O\), at a point \(H'\), the coordinates of which are
\[ H' = \begin{cases} X_2 = U_2 - U_1(S_{h2}/S_{h1}) \\ X_3 = U_3 - U_1(S_{h3}/S_{h1}) \end{cases}. \] (25)

The magnification ratio is simply the ratio of the lengths of the vectors \(OH'\) and \(OH\), the latter being by definition equal to 1:
\[ M = \left( \frac{U_2 - U_1S_{h2}}{S_{h1}} \right)^2 + \left( \frac{U_3 - U_1S_{h3}}{S_{h1}} \right)^2 \right]^{1/2}. \] (26)

V. Projection of a crystallographic plane other than the reflecting plane: projected width

The trace, on the detector, of a crystallographic plane of normal \([pqr]\) is determined by the intersection of the detector plane and the plane containing the trace of the \((pqr)\) plane on the crystal surface and \(S_h\).

Let us call \(\mathbf{p}\) a unit vector, parallel to the direction \([pqr]\), the plane normal, expressed in the crystal axes.

The equation of the \((pqr)\) plane, passing through the origin, is
\[ X_1P_1 + X_2P_2 + X_3P_3 = 0, \] (27)

where
\[ (\mathbf{P}') = (A)(\mathbf{p}). \]

The sample normal \(\mathbf{v}\) is defined by
\[ \begin{cases} v_1 = 1 \\ v_2 = 0 \\ v_3 = 0. \end{cases} \] (28)

The trace of the \((pqr)\) plane in the sample surface is parallel to the vector \(\mathbf{T}\) defined by
\[ \mathbf{T} = \begin{cases} T_1 = 0 \\ T_2 = -P_3 \\ T_3 = P_2. \end{cases} \] (29)

As in the former paragraph, the equation of the projection plane is
\[ \begin{vmatrix} X_1 & X_2 & X_3 \\ S_{h1} & S_{h2} & S_{h3} \\ T_1 & T_2 & T_3 \end{vmatrix} = \text{constant}, \] (30)

where \(S_h\) is given by (16).

\[ \mathbf{T} = (C)(B)(\mathbf{T}'). \] (31)

The angle \(\eta\) between the trace of the projection plane in the detector plane and the $\xi_2$ axis is (Fig. 4)
\[ \eta = \arctan \left( \frac{S_{h3}T_1 - S_{h1}T_3}{S_{h1}T_2 - S_{h2}T_1} \right). \] (32)

In order to compute the projected width of the \((pqr)\) plane, assuming a parallel-sided sample, the constant in
(30) has to be adjusted in the following way: the (pqr)
plane (equation 27) intercepts the crystal surfaces
\[ X_1 = \pm t, \]
\[ 2t \text{ being the crystal thickness, along traces defined by} \]
\[ \text{surface } a \begin{cases} X'_1 = -t \\ -tP_1 + X'_2P_2 + X'_3P_3 = 0 \end{cases} \]
\[ \text{surface } b \begin{cases} X'_1 = +t \\ tP_1 + X'_2P_2 + X'_3P_3 = 0 \end{cases}. \]

Now, the constant in (30) should be adjusted in order that (30) represents either a plane containing the trace (33a) or the trace (33b). Equation (30) will do so provided the constant is such that the projection planes contain a point on trace (33a) and a point on trace (33b). The choice of these points is arbitrary provided their coordinates remain finite. The projection planes corresponding to the traces (33a) and (33b) intersect the detector planes along two straight lines of general equations:
\[ \begin{cases} a: \alpha \xi_2 + \beta \xi_3 + \gamma_a = 0 \\ b: \alpha \xi_2 + \beta \xi_3 + \gamma_b = 0 \end{cases} \]

The distances from the origin \( O' \) to those straight lines are
\[ \begin{cases} a: \delta_a = \gamma_a/\sqrt{\alpha^2 + \beta^2} \\ b: \delta_b = \gamma_b/\sqrt{\alpha^2 + \beta^2} \end{cases}. \]

The projected width of the (pqr) plane is therefore
\[ \omega = |\delta_a - \delta_b| = |\gamma_a - \gamma_b|/\sqrt{\alpha^2 + \beta^2}. \]

Finally, for a \([uvw]\) direction contained in the (pqr)
plane \((uv + vq + wr = 0)\), \(\psi, M, \eta\) and \(\omega\) are linked by the relation
\[ M \sin |\psi - \eta| = \omega \cos \epsilon, \]
where \(\epsilon\) is the angle between the \([uvw]\) direction and the sample normal. This relation may be used for cross-checking the results.

**VI. Application**

The example described hereafter is borrowed from a
study of the plastic behaviour of Fe-Si single crystals, which is pursued in collaboration with D. K. Bowen from Warwick University, England.

The sample (Fig. 5a) is [100] oriented and the tensile axis is [011], an orientation meant to favour double glide. The geometry of the sample and the detector with respect to the incident beam is illustrated in Fig. 5(b), (c); the horizontal direction in the specimen is 5° from [010]. This geometry was imposed by:

(i) our wish to use the 002 reflection as the main spot;
(ii) constraints due to the lack of space which impeded an exact 002 reflection main-spot geometry.

Fig. 6 shows the 002 and 0T1 spots in the unstrained state. The operative wavelengths are 1.79 and 1.95 Å for the 002 and 0T1 spots, respectively. Although the 0T1 reflection is, at a given wavelength, more intense than the 002 reflection, both spots have, in the present experiment, comparable background diffraction intensities due to the difference in absorption at 1.79 and 1.95 Å.

Magnetic domain walls are visible in the topographs: (011) and (011) 90° walls are visible in the 002 reflection whereas only (011) 90° walls are visible in the 0T1 reflection (Polcarová & Kaczér, 1967). The intersections of those domain walls with the crystal surfaces provide fairly accurate {011} directions in the sample surface (provided the sample normal is precisely [100]) and allow for a proper comparison of computed projected directions with experimental data. The analysis may be extended to the [010] and [001] directions since, although {001} 180° walls are not visible due to the lack of magnetostrictive distortions, their junctions with {011} 90° walls may always be detected as narrow kinks in the 90° walls. Fig. 7 shows for instance an enlargement of Fig. 6(a) together with a sketch of the magnetic domain configuration: relevant {011} directions are indicated.

Fig. 8 shows a reconstruction of the image shape taking into account computed [011] and [0T1] projected directions as well as magnification ratios along those directions, the width of the sample and the beam height being known. Projected ⟨011⟩, ⟨001⟩ and ⟨111⟩ directions of interest are indicated in the figure. The comparison of Fig. 6 and Fig. 8, substantiated by Fig. 7 shows that the agreement between computed and observed ⟨001⟩ and ⟨011⟩ projected directions is excellent. Further, the ratios of the lengths, in the 002 and 0T1 reflections, of wall segments fully contained
within a subgrain are in full agreement with computed values. Care should be taken when comparing the exact-image dimensions with reconstructed images for crystals containing subgrains as the wavelengths, hence the Bragg angles pertaining to the various subgrains differ, leading to subgrain images which either overlap or are separated by a gap (Hart, 1975). This effect, however, leads to only minor overall image dilatation or contraction, provided the disorientation between subgrains as well as the distance between sample and detector remains small.

Checking the computed values of the projected widths of planes is a more delicate task. In Fig. 6(b), (01I) 90° walls such as W exhibit a measurable projected width. It is obvious, however, that the projected widths of the various wall segments vary drastically, implying that these walls have varying inclinations with respect to the sample surface. It should be noted that the (01I) wall-segment normals should belong to the [011] zone in order to prevent a build-up of magnetic charges. Hence, the intersections of those walls with the [100] surface remain [011].

The comparison between computed and measured projected widths of crystallographic planes therefore appears problematic if those planes are to be materialized by domain walls. However, the computed projected widths of any plane and magnification ratios along directions contained in that plane have been linked via (37). A perfect agreement has already been found, providing a fair confidence in the calculations.

Finally, Fig. 8 shows that the [111] and [111] directions have markedly different projection characteristics in the 002 and 011 reflections. These directions should therefore easily be distinguished.

Fig. 9 shows the 002 and the 011 spots of the sample shown in Fig. 6 after plastic deformation. Owing to the orientation of the tensile axis, the two most highly stressed slip systems are (211)[111] and (211)[111] with equal Schmid factors (\(S = 0.47\)) (see Fig. 5a). The intersections of the (211) and (211) planes with sample surfaces are identical and parallel to [011]. Therefore, those glide systems may not easily be differentiated.

Slip is seen to occur in small glide elements such as A in Fig. 9(a). Owing to the orientations of the [111] and [111] directions with respect to the surface of the tensile specimen, these elements are anticipated to be bound by screw dislocations. They should therefore be bound by dislocations lying along the [111] or [111] directions according to the slip system. It is clear, by comparing Fig. 9(a), (b) and Fig. 8(a), (b), that glide elements such as A are bound by dislocations whose average direction lies close to [111]. Conversely, glide elements such as B

Fig. 9. White-beam Laue topographs of the tensile specimen after plastic deformation: \(\sigma = 270\) MPa. (a) 002 fundamental reflection; (b) 011 fundamental reflection. The experimental conditions are identical to those of Fig. 6.
are bound by dislocations of average [111] direction. The two slip systems may now therefore easily be distinguished and to some extent characterized.

Conclusion

The calculations presented in this paper may serve to:

(i) characterize the observed spots (operative wavelength, diffraction vector);
(ii) provide a graphic representation of the image distortions;
(iii) index the observed orientations of unidentified objects contained in the sample.

This last possibility is probably the most interesting as most of the geometrical information which is needed when analysing images is contained in the various spots recorded on a single photographic plate in a white-beam X-ray topography experiment.

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APPENDIX

Calculations pertaining to the specific geometry illustrated in Fig. 2

For this configuration (B) is given by

\[
\mathbf{B} = \begin{pmatrix}
\cos \gamma & 0 & \sin \gamma \\
0 & 1 & 0 \\
-\sin \gamma & 0 & \cos \gamma
\end{pmatrix}.
\]

For the sake of brevity, let us write \( \theta_{hkl} = \theta \).

I. Position of the spots

One gets

\[
\begin{align*}
N'_1 &= \cos \gamma N_1 - \sin \gamma N_3 \\
N'_2 &= N_2 \\
N'_3 &= \sin \gamma N_1 + \cos \gamma N_3
\end{align*}
\] (A1)

from (13),

\[
\begin{align*}
S'_h = & \begin{pmatrix}
\cos 2\theta \\
\sin 2\theta (A/B) \\
\sin 2\theta (C/B)
\end{pmatrix} \\
S_h = & \begin{pmatrix}
\cos 2\gamma \cos 2\theta + \sin 2\gamma \sin 2\theta (C/B) \\
\sin 2\theta (A/B) \\
-\sin 2\gamma \cos 2\theta + \cos 2\gamma \sin 2\theta (C/B)
\end{pmatrix}
\] (A2)

from (14) where \( A = N_2, B = (N_2^2 + N_3^2)^{1/2} = N_3^2 \) and \( C = N_3^2 \).

from (16). Finally, one has

\[
\begin{align*}
\xi_2 &= -\frac{d}{B \cos 2\gamma \cos 2\theta + C \sin 2\gamma \sin 2\theta} \\
\xi_3 &= \frac{d}{C \sin 2\theta}
\end{align*}
\] (A4)

For the main spot, one gets \( \xi_2 = 0, \xi_3 = d \tan 2\theta \) as expected.

II. Trace of the plane of incidence

From (19), one gets

\[
\psi = \arctan \left( -\frac{C}{A \cos 2\gamma} \right).
\] (A5)

For the main spot, \( A = 0 \) and \( |\psi| = \pi/2 \).

III. Projection of a crystallographic direction

One gets

\[
\begin{align*}
U_1 &= U_1 \cos \gamma + U_3 \sin \gamma \\
U_2 &= U_2 \\
U_3 &= -U_1 \sin \gamma + U_3 \cos \gamma
\end{align*}
\] (A6)

from (21) and

\[
\psi = \arctan \left\{ \left[ U_1 (C \cos \gamma \sin 2\theta - B \sin \gamma \cos 2\theta) \right] \\
- U_3 (C \sin \gamma \sin 2\theta + B \cos \gamma \cos 2\theta) \\
\times \left[ U_2 (C \cos 2\gamma \sin 2\theta + B \cos 2\gamma \cos 2\theta) \right] \\
- A \sin 2\theta (U_1 \cos \gamma + U_3 \sin \gamma) \right\} \] (A7)

from (22). Equation (A7) shows, as expected, that the direction satisfying \( U_1 = U_3 = 0, U_2 = 1 \) projects along the \( \xi_2 \) axis for any spot. For the main spot, the direction satisfying \( U_1 = U_3 = 0, U_2 = 1 \) projects along \( \xi_3 \), as anticipated, since \( U_2 = 0 \) and \( A = 0 \).

The magnification ratio (equation 26) along the \( [uvw] \) direction should for the main spot be equal to 1 for the direction defined by \( U_1 = U_3 = 0, U_2 = 1 \), and equal to \( \cos \theta \) for the direction defined by \( U_1 = U_2 = 0, U_3 = 1 \). These results may easily be checked.

IV. Projection of a crystallographic plane – projected width

Equation (31) now reads

\[
\begin{align*}
T_1 &= P_2 \sin \gamma \\
T_2 &= -P_3 \\
T_3 &= P_2 \cos \gamma
\end{align*}
\] (A8)

and (32) becomes

\[
\eta = \arctan \left\{ \left[ P_2 (B \cos \gamma \cos 2\theta + C \sin \gamma \sin 2\theta) \right] \\
\times \left[ B P_3 \cos 2\gamma \cos 2\theta + \sin 2\theta \right] \\
\times \left[ A P_2 \sin \gamma + C P_3 \sin 2\gamma \right] \right\} \] (A9)

Equation (A9) shows, as expected, that the trace on the crystal surface of the plane satisfying \( P_1 = P_3 = 0, P_2 = 1 \) projects along the \( \xi_2 \) axis since \( P_2 = 0 \) for any spot. For the main spot, the trace on the crystal surface of the plane satisfying \( P_1 = P_3 = 0, P_2 = 1 \) projects along the \( \xi_3 \) axis as anticipated, since \( P_3 = 0 \) and \( A \) are both equal to zero.

Let us finally consider two points, the coordinates of which are
They satisfy (33a) and (33b), respectively. This choice is good as long as $P_3$ is different from 0.

Transformation in the $X_i$ set of coordinates yields

$$X_i' = -X_i = -t \cos \gamma + t \sin \gamma \frac{P_1}{P_3},$$

$$X_2' = X_2 = 0,$$

$$X_3' = -X_3 = t \sin \gamma + t \cos \gamma \frac{P_1}{P_3}.$$  \hspace{1cm} (A10a)

Equation (30) now reads

$$\begin{vmatrix} X_1 - X_1' & X_2 - X_2' & X_3 - X_3' \\ S_{h_1} & S_{h_2} & S_{h_3} \\ T_1 & T_2 & T_3 \end{vmatrix} = 0$$  \hspace{1cm} (A12)

for surface (a);

$$\begin{vmatrix} X_1 - X_1' & X_2 - X_2' & X_3 - X_3' \\ S_{h_1} & S_{h_2} & S_{h_3} \\ T_1 & T_2 & T_3 \end{vmatrix} = 0$$  \hspace{1cm} (A13)

for surface (b). Transformation into the $\xi_i$ set of coordinates is straightforward. The planes defined by (A12) and (A13) intersect the detector plane $\xi_1 = 0$ along two lines. The distance between those straight lines is the $(pqr)$ plane projected width.

Let us call

$$E = S_{h_2} T_3 - S_{h_3} T_2 = -P_3 \sin 2\gamma \cos 2\theta + \left[\frac{(A/B)P_2}{P_3} \cos \gamma + \frac{(C/B)P_3}{P_3} \cos 2\gamma\right] \sin 2\theta$$

$$F = \alpha = S_{h_3} T_1 - S_{h_1} T_3 = -P_2 \left[\cos \gamma \cos 2\theta + \frac{(C/B)}{P_3} \sin \gamma \sin 2\theta\right]$$

$$G = \beta = S_{h_1} T_2 - S_{h_2} T_1 = -\left[P_3 \cos 2\gamma \cos 2\theta + \frac{(A/B)}{P_3} \sin \gamma + \frac{(C/B)}{P_3} \sin 2\gamma \sin 2\theta\right].$$

According to (34) and (36), one gets

$$\omega = 2t \frac{|E(P_3 \cos \gamma - P_1 \sin \gamma) - G(P_1 \cos \gamma + P_3 \sin \gamma)|}{P_3(F^2 + G^2)^{1/2}}$$  \hspace{1cm} (A15)

For the main spot ($A = 0, B = C = \cos \gamma = \cos \theta$) and for the plane satisfying $P_1 = P_2 = 0, P_3 = 1$, i.e. the reflecting plane, (A15) reduces to $\omega = 2t \sin \theta$ as anticipated. If $P_3 = 0$, another choice of points needs to be made. The nature of the calculation remains, however, the same.

References


