Appendix A: Mathematical details

The functions $f_i(x, y, \varrho_1, \varrho_2)$ in Eq. 7 are given by

$$f_1(x, y, \varrho_1, \varrho_2) = \frac{1}{2}(\varrho_1 - \varrho_2)^2 \tag{A1a}$$
$$f_2(x, y, \varrho_1, \varrho_2) = -\frac{1}{2}(\varrho_1 - \varrho_2)^2 \cos(2x) \tag{A1b}$$
$$f_3(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)^2 x \sin(2x) \tag{A1c}$$
$$f_4(x, y, \varrho_1, \varrho_2) = \frac{1}{2}(\varrho_1 - \varrho_2)^2 x^2 \tag{A1d}$$
$$f_5(x, y, \varrho_1, \varrho_2) = \frac{1}{2}(\varrho_1 - \varrho_2)^2 x^2 \cos(2x) \tag{A1e}$$
$$f_6(x, y, \varrho_1, \varrho_2) = \frac{1}{2}\varrho_2^2 \tag{A1f}$$
$$f_7(x, y, \varrho_1, \varrho_2) = -\frac{1}{2}\varrho_2^2 \cos(2x)\cos(2y) \tag{A1g}$$
$$f_8(x, y, \varrho_1, \varrho_2) = \frac{1}{2}\varrho_2^2 \sin(2x)\sin(2y) \tag{A1h}$$
$$f_9(x, y, \varrho_1, \varrho_2) = -\varrho_2^2 x \sin(2x)\cos(2y) \tag{A1i}$$
$$f_{10}(x, y, \varrho_1, \varrho_2) = -\varrho_2^2 x \cos(2x)\sin(2y) \tag{A1j}$$
$$f_{11}(x, y, \varrho_1, \varrho_2) = -\varrho_2^2 \sin(2x)\, y \cos(2y) \tag{A1k}$$
$$f_{12}(x, y, \varrho_1, \varrho_2) = -\varrho_2^2 \cos(2x)\, y \sin(2y) \tag{A1l}$$
$$f_{13}(x, y, \varrho_1, \varrho_2) = \frac{1}{2}\varrho_2^2 x^2 \tag{A1m}$$
$$f_{14}(x, y, \varrho_1, \varrho_2) = \frac{1}{2}\varrho_2^2 y^2 \tag{A1n}$$
$$f_{15}(x, y, \varrho_1, \varrho_2) = \varrho_2^2 xy \tag{A1o}$$
$$f_{16}(x, y, \varrho_1, \varrho_2) = \frac{1}{2}\varrho_2^2 x^2 \cos(2x)\cos(2y) \tag{A1p}$$
$$f_{17}(x, y, \varrho_1, \varrho_2) = -\frac{1}{2}\varrho_2^2 x^2 \sin(2x)\sin(2y) \tag{A1q}$$
$$f_{18}(x, y, \varrho_1, \varrho_2) = \frac{1}{2}\varrho_2^2 \cos(2x)\, y^2 \cos(2y) \tag{A1r}$$
$$f_{19}(x, y, \varrho_1, \varrho_2) = -\frac{1}{2}\varrho_2^2 \sin(2x)\, y^2 \sin(2y) \tag{A1s}$$
$$f_{20}(x, y, \varrho_1, \varrho_2) = \varrho_2^2 x \cos(2x)\, y \cos(2y) \tag{A1t}$$
$$f_{21}(x, y, \varrho_1, \varrho_2) = -\varrho_2^2 x \sin(2x)\, y \sin(2y) \tag{A1u}$$
$$f_{22}(x, y, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cos(y) \tag{A1v}$$
$$f_{23}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 \cos(2x)\cos(y) \tag{A1w}$$
$$f_{24}(x, y, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \sin(2x)\sin(y) \tag{A1x}$$
$$f_{25}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 x \sin(2x)\cos(y) \tag{A1y}$$
$$f_{26}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 \sin(2x)\, y \cos(y) \tag{A1z}$$
$$f_{27}(x, y, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 x \sin(y) \tag{A1aa}$$
$$f_{28}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 x \cos(2x)\sin(y) \tag{A1bb}$$
$$f_{29}(x, y, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 y \sin(y) \tag{A1cc}$$
$$f_{30}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 \cos(2x)\, y \sin(y) \tag{A1dd}$$
$$f_{31}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 x \sin(2x)\cos(y) \tag{A1ee}$$
$$f_{32}(x, y, \varrho_1, \varrho_2) = -(\varrho_1 - \varrho_2)\,\varrho_2 x \sin(y) \tag{A1ff}$$
$$\tag{A1gg}$$

$$f_{33}(x,y,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)\varrho_2 x\cos\left(2x\right)\sin\left(y\right) \tag{A1hh}$$

$$f_{34}(x,y,\varrho_1,\varrho_2) = \left(\varrho_1-\varrho_2\right)\varrho_2 x^2\cos\left(y\right) \tag{A1ii}$$

$$f_{35}(x,y,\varrho_1,\varrho_2) = \left(\varrho_1-\varrho_2\right)\varrho_2 x^2\cos\left(2x\right)\cos\left(y\right) \tag{A1jj}$$

$$f_{36}(x,y,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)\varrho_2 x^2\sin\left(2x\right)\sin\left(y\right) \tag{A1kk}$$

$$f_{37}(x,y,\varrho_1,\varrho_2) = \left(\varrho_1-\varrho_2\right)\varrho_2 xy\cos\left(y\right) \tag{A1ll}$$

$$f_{38}(x,y,\varrho_1,\varrho_2) = \left(\varrho_1-\varrho_2\right)\varrho_2 x\cos\left(2x\right)y\,\cos\left(y\right) \tag{A1mm}$$

$$f_{39}(x,y,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)\varrho_2 x\sin\left(2x\right)y\sin\left(y\right). \tag{A1nn}$$

The factors F_i appearing in the Laplace transforms of these functions are

$$F_1(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\left(\varrho_1-\varrho_2\right)^2 \tag{A2a}$$

$$F_2(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\frac{1}{2}\left(\varrho_1-\varrho_2\right)^2\cdot L_c(s,Z_R,0,2) \tag{A2b}$$

$$F_3(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)^2\cdot L_s(s,Z_R,1,2) \tag{A2c}$$

$$F_4(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\left(\varrho_1-\varrho_2\right)^2\cdot L(s,Z_R,2) \tag{A2d}$$

$$F_5(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\left(\varrho_1-\varrho_2\right)^2\cdot L_c(s,Z_R,2,2) \tag{A2e}$$

$$F_6(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\varrho_2^2 \tag{A2f}$$

$$F_7(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\frac{1}{2}\varrho_2^2\cdot L_c(s,Z_R,0,2)\cdot L_c(t,Z_\Delta,0,2) \tag{A2g}$$

$$F_8(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\varrho_2^2\cdot L_s(s,Z_R,0,2)\cdot L_s(t,Z_\Delta,0,2) \tag{A2h}$$

$$F_9(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\varrho_2^2\cdot L_s(s,Z_R,1,2)\cdot L_c(t,Z_\Delta,0,2) \tag{A2i}$$

$$F_{10}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\varrho_2^2\cdot L_c(s,Z_R,1,2)\cdot L_s(t,Z_\Delta,0,2) \tag{A2j}$$

$$F_{11}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\varrho_2^2\cdot L_s(s,Z_R,0,2)\cdot L_c(t,Z_\Delta,1,2) \tag{A2k}$$

$$F_{12}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\varrho_2^2\cdot L_c(s,Z_R,0,2)\cdot L_s(t,Z_\Delta,1,2) \tag{A2l}$$

$$F_{13}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\varrho_2^2\cdot L(s,Z_R,2) \tag{A2m}$$

$$F_{14}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\varrho_2^2\cdot L(t,Z_\Delta,2) \tag{A2n}$$

$$F_{15}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \varrho_2^2\cdot L(s,Z_R,1)\cdot L(t,Z_\Delta,1) \tag{A2o}$$

$$F_{16}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\varrho_2^2\cdot L_c(s,Z_R,2,2)\cdot L_c(t,Z_\Delta,0,2) \tag{A2p}$$

$$F_{17}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\frac{1}{2}\varrho_2^2\cdot L_s(s,Z_R,2,2)\cdot L_s(t,Z_\Delta,0,2) \tag{A2q}$$

$$F_{18}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \frac{1}{2}\varrho_2^2\cdot L_c(s,Z_R,0,2)\cdot L_c(t,Z_\Delta,2,2) \tag{A2r}$$

$$F_{19}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\frac{1}{2}\varrho_2^2\cdot L_s(s,Z_R,0,2)\cdot L_s(t,Z_\Delta,2,2) \tag{A2s}$$

$$F_{20}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \varrho_2^2\cdot L_c(s,Z_R,1,2)\cdot L_c(t,Z_\Delta,1,2) \tag{A2t}$$

$$F_{21}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\varrho_2^2\cdot L_s(s,Z_R,1,2)\cdot L_s(t,Z_\Delta,1,2) \tag{A2u}$$

$$F_{22}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \left(\varrho_1-\varrho_2\right)\varrho_2\cdot L_c(t,Z_\Delta,0,1) \tag{A2v}$$

$$F_{23}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)\varrho_2\cdot L_c(s,Z_R,0,2)\cdot L_c(t,Z_\Delta,0,1) \tag{A2w}$$

$$F_{24}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = \left(\varrho_1-\varrho_2\right)\varrho_2\cdot L_s(s,Z_R,0,2)\cdot L_s(t,Z_\Delta,0,1) \tag{A2x}$$

$$F_{25}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)\varrho_2\cdot L_s(s,Z_R,1,2)\cdot L_c(t,Z_\Delta,0,1) \tag{A2y}$$

$$F_{26}(s,Z_R,t,Z_\Delta,\varrho_1,\varrho_2) = -\left(\varrho_1-\varrho_2\right)\varrho_2\cdot L_s(s,Z_R,0,2)\cdot L_c(t,Z_\Delta,1,1) \tag{A2z}$$

(A2aa)

$$F_{27}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cdot L(s, Z_R, 1) \cdot L_s(t, Z_\Delta, 0, 1) \tag{A2bb}$$
$$F_{28}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_c(s, Z_R, 1, 2) \cdot L_s(t, Z_\Delta, 0, 1) \tag{A2cc}$$
$$F_{29}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_s(t, Z_\Delta, 1, 1) \tag{A2dd}$$
$$F_{30}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_c(s, Z_R, 0, 2) \cdot L_s(t, Z_\Delta, 1, 1) \tag{A2ee}$$
$$F_{31}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_s(s, Z_R, 1, 2) \cdot L_c(t, Z_\Delta, 0, 1) \tag{A2ff}$$
$$F_{32}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L(s, Z_R, 1) \cdot L_s(t, Z_\Delta, 0, 1) \tag{A2gg}$$
$$F_{33}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_c(s, Z_R, 1, 2) \cdot L_s(t, Z_\Delta, 0, 1) \tag{A2hh}$$
$$F_{34}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cdot L(s, Z_R, 2) \cdot L_c(t, Z_\Delta, 0, 1) \tag{A2ii}$$
$$F_{35}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_c(s, Z_R, 2, 2) \cdot L_c(t, Z_\Delta, 0, 1) \tag{A2jj}$$
$$F_{36}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_s(s, Z_R, 2, 2) \cdot L_s(t, Z_\Delta, 0, 1) \tag{A2kk}$$

$$F_{37}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cdot L(s, Z_R, 1) \cdot L_c(t, Z_\Delta, 1, 1) \tag{A2ll}$$
$$F_{38}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = (\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_c(s, Z_R, 1, 2) \cdot L_c(t, Z_\Delta, 1, 1) \tag{A2mm}$$
$$F_{39}(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = -\,(\varrho_1 - \varrho_2)\,\varrho_2 \cdot L_s(s, Z_R, 1, 2) \cdot L_s(t, Z_\Delta, 1, 1). \tag{A2nn}$$

with the functions $L(s, Z, n)$, $L_s(s, Z, n, \nu)$ and $L_c(s, Z, n, \nu)$ defined in Eq. 16.

For the normalization to the forward scattering (Eq. 18) the contributions

$$n_1(R, \Delta, \varrho_1, \varrho 2) = \varrho_1^2 R^6 \tag{A3a}$$
$$n_2(R, \Delta, \varrho_1, \varrho 2) = 6\varrho_1\varrho_2 R^5 \Delta \tag{A3b}$$
$$n_3(R, \Delta, \varrho_1, \varrho 2) = (6\varrho_1\varrho_2 + 9\varrho_2^2) R^4 \Delta^2 \tag{A3c}$$
$$n_4(R, \Delta, \varrho_1, \varrho 2) = (2\varrho_1\varrho_2 + 18\varrho_2^2) R^3 \Delta^3 \tag{A3d}$$
$$n_5(R, \Delta, \varrho_1, \varrho 2) = 15\varrho_2^2 R^2 \Delta^4 \tag{A3e}$$
$$n_6(R, \Delta, \varrho_1, \varrho 2) = 6\varrho_2^2 R \Delta^5 \tag{A3f}$$
$$n_7(R, \Delta, \varrho_1, \varrho 2) = \varrho_2^2 \Delta^6 \tag{A3g}$$

appear. In the Laplace transform of the forward scattering in Eq. 19 the contributions K_i are given by

$$K_1(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = \varrho_1^2 L(s, Z_R, 6) \tag{A4a}$$
$$K_2(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = 6\varrho_1\varrho_2 L(s, Z_R, 5) L(t, Z_\Delta, 1) \tag{A4b}$$
$$K_3(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = \left(6\varrho_1\varrho_2 + 9\varrho_2^2\right) L(s, Z_R, 4) L(t, Z_\Delta, 2) \tag{A4c}$$
$$K_4(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = \left(2\varrho_1\varrho_2 + 18\varrho_2^2\right) L(s, Z_R, 3) L(t, Z_\Delta, 3) \tag{A4d}$$
$$K_5(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = 15\varrho_2^2 L(s, Z_R, 2) L(t, Z_\Delta, 4) \tag{A4e}$$
$$K_6(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = 6\varrho_2^2 L(s, Z_R, 1) L(t, Z_\Delta, 5) \tag{A4f}$$
$$K_7(s, Z_R, t, Z_\Delta, \varrho_1, \varrho_2) = \varrho_2^2 L(t, Z_\Delta, 6). \tag{A4g}$$

The C-code is available at **URL:** *www.wagner.chemie.uni-rostock.de.*

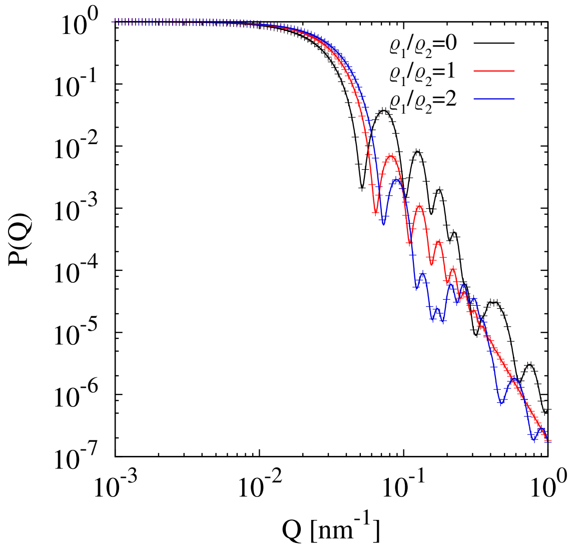

FIG. 1 Comparison of numeric results employing an adaptive Gauss-Kronrod integrator (crosses) with the analytic result for core-shell particles with a core diameter of $\sigma_0 = 2R_0 = 100\,\mathrm{nm}$, and a shell thickness of $\Delta_0 = 20\,\mathrm{nm}$. The parameters $Z_R = 300$ and $Z_\Delta = 200$ correspond to relative polydispersities of the core diameter of $\mathcal{P}_c = 0.058$ and the shell thickness of $\mathcal{P}_c = 0.071$, respectively. The ratio $\varrho_1/\varrho_2 = 0$ corresponds to a hollow sphere, $\varrho_1/\varrho_2 = 1$ corresponds to a homogeneous sphere and finally $\varrho_1/\varrho_2 = 2$ to a core-shell particle with a scattering length density of the core twice the scattering length density of the shell.

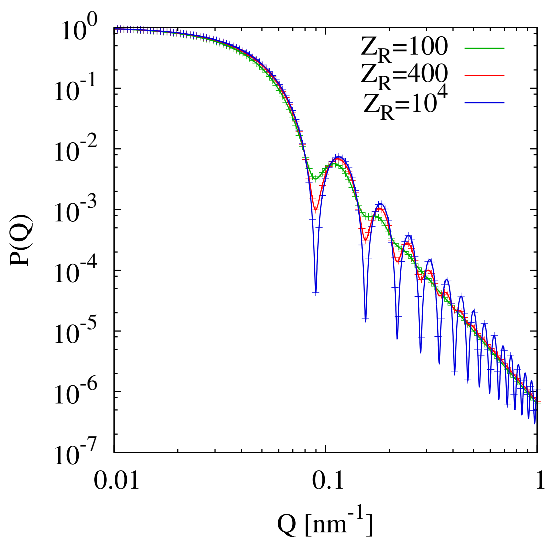

FIG. 2 Form factors for particles with $\varrho_2 = 0$, i.e., homogeneous spheres with the core size $\sigma_0 = 100\,\mathrm{nm}$. Due to avoiding large exponents and large arguments to the Γ-function also results for extremely small polydispersities even less than $\mathcal{P}_c = 10^{-4}$, i.e., $Z_R > 10^8$ does not lead to numerical overflow. For small polydispersities the analytic implementation is even more stable than the numerical one.

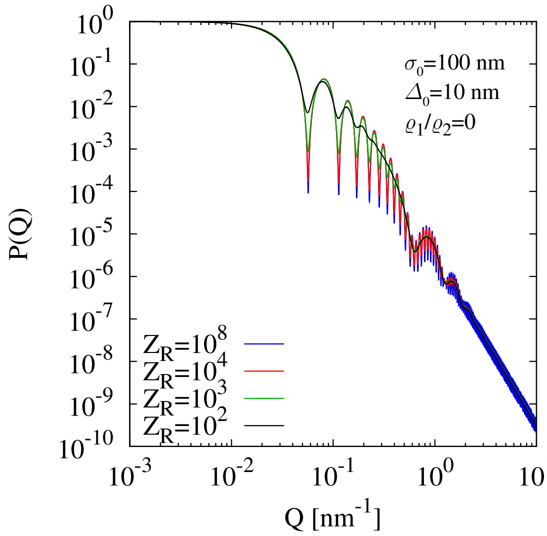

FIG. 3 Form factor of hollow spheres ($\varrho_1/\varrho_2 = 0$) with the inner diameter $\sigma_0 = 100\,\mathrm{nm}$ and the shell thickness of $\Delta_0 = 10\,\mathrm{nm}$. The reduced polydispersity of the shell thickness is kept constant at $\mathcal{P}_\Delta = 0.1$ ($Z_\Delta = 100$), whereas the polydispersity of the core size varies from $\mathcal{P}_\sigma = 0.1$ to $\mathcal{P}_\sigma = 10^{-4}$ ($Z_R = 100$ to $Z_R = 10^8$).

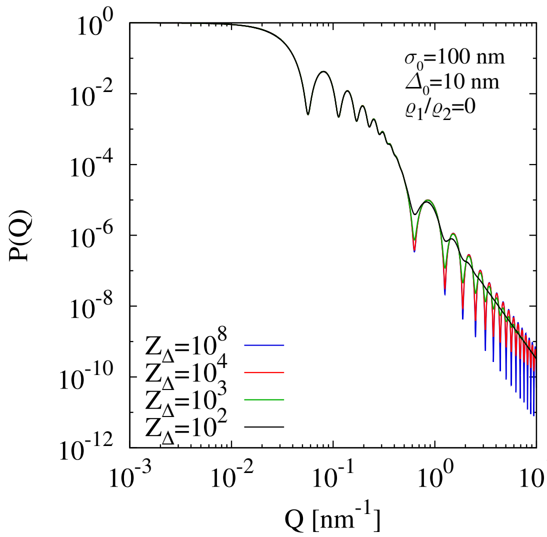

FIG. 4 Form factor of hollow spheres ($\varrho_1/\varrho_2 = 0$) with the inner diameter $\sigma_0 = 100\,\mathrm{nm}$ and the shell thickness of $\Delta_0 = 10\,\mathrm{nm}$. The reduced polydispersity of the inner diameter is kept constant at $\mathcal{P}_\sigma = 0.1$ ($Z_R = 100$), whereas the polydispersity of the shell thickness varies from $\mathcal{P}_\Delta = 0.1$ to $\mathcal{P}_\Delta = 10^{-4}$ ($Z_\Delta = 100$ to $Z_\Delta = 10^8$).