Book Reviews

Works intended for notice in this column should be sent direct to the Editor (A. J. C. Wilson, Department of Physics, University College, Cathays Park, Cardiff, Great Britain). As far as practicable books will be reviewed in a country different from that of publication.

A Class-room Method for the Derivation of the 230 Space Groups. By N. V. Belov. Translated by V. Balashov from Trudy Inst. Krist. Akad. Nauk. SSSR, No. 6, 25–62 (1951). Appearing as Proc. Leeds Phil. Lit. Soc., Sci. Sect. VIII, Pt I. Pp. ii+46, with 29 Figs. (Austick's Bookshops, Woodhouse Lane, Leeds 2, England). Price 7*s* 6*d*.

The traditional textbook derivation of the space groups starts with a derivation or presentation of the crystallographic point groups. These groups are divided into (generally) seven systems. Before, or after, having presented the point groups and systems a derivation or presentation is given of the possible translatory repetitions of points in the different systems, all points having the same environment in the same orientation. Independent and product action of point group and translatory operators (not only *lattice* translations) are then envisaged. At the same time the condition that any repeated point should have the same environment in *parallel* orientation is cancelled: change of orientation and change of 'hand' of environment is admitted.

In the derivation of point- and space-group operations as well as in the systematization, symmetry *axes* generally play a primary rôle: even operations of the second kind are described in terms of inversion (or alternating) *axes*. The *strict* derivation of all symmetry operations that are possible is generally given in a complicated way, if at all.

An elementary way of deriving all possible (crystallographic) point-group operations was given by C. Viola (*Z. Kristallogr.* **27**, 1, 1896). He chose as his basic operation one of the *second* kind (which is of course necessary if a single basic operation is used) and proved that it is sufficient to consider *no* reflexion (i.e. operation 1), *one* reflexion (operation $\bar{2}$), the product of *two* reflexions (intersecting planes) (operation n) and the product of *three* reflexions, the planes intersecting in one point (operation $\bar{n}$). (If planes are allowed to coalesce, the operations of none and one reflexion are special cases of two and three reflexions.) It is possible to develop Viola's strategy to space operations and prove that it is then necessary and sufficient to consider operations which can be expressed as products of (none, one, two, and) three and four reflexions where the planes do not generally pass through one point.

Belov bases his work on papers by G. V. Wulff (*Proc. Warsaw Nat. Hist. Soc.* 1896, 1897) and—according to the translator—the paper by C. Viola. (Only the translation will be considered here. The papers by Wulff have not been available to the reviewer but seem related to the Viola paper.) Belov stresses 'the fundamental importance of the symmetry plane' but does not take the full step to work exclusively with the operations of successive reflexions. Instead he uses as fundamental operations lattice translations (in terms of reflexions, the product of two 'parallel' reflexions) and the operations of symmetry planes (the product of three reflexions where two mirrors intersect in a line parallel to or in the remaining plane). These operations are first considered as *independent*—in what Belov calls 'senior classes'. Their products are found as subgroup operations—'babies' in the 'junior' classes. Belov does not consider if this is allowable, and it is as a matter of fact not strictly allowable. The operation of *one* mirror plane or the product operation of *two parallel* mirror planes are, as mentioned, included among Belov's fundamental operations. The product of *two intersecting* mirror planes (a rotation axis) might with Belov's fundamental operations come out as the product of two symmetry planes (intersecting parallel to the produced axis) with no axial translatory components or cancelling components. However, these symmetry planes cannot always exist independently. This is the case for planes generating the two-fold axes in the space groups of the classes 2 and $2/m$ (unless the translation group is first specialized). Similarly with the two-fold axes in the space group $R32$ and with the three-fold axes in the primitive cubic space groups with non-intersecting rotatory triads ($P2_13$, $Pa3$, $P4_132$, $P4_332$). The product of *three* (or any higher odd number of) reflexions can be shown to be equivalent either with one reflexion (glide or not), which is considered as fundamental by Belov, or the operation of a rotatory inversion axis. The axis $\bar{3}$ in rhombohedral cells can evidently not be created by independent symmetry planes. The only new operation produced by *four* (or any higher even number of reflexions) is generally described in terms of a screw axis. We shall not go into details but it immediately follows that if an intrinsically enantiomorphous element (3_1, 3_2, 4_1, 4_3, 6_1, 6_2, 6_4, or 6_5) does come out at all as the product of independent symmetry planes (i.e. of operations of the second kind) it must appear with both parities (3_1 and 3_2, etc.).

We cannot here deal with the several auxiliary methods Belov uses to find missing space groups.

It seems to the reviewer that the derivations given by Belov would be easier to follow if certain underlying concepts and hypotheses had been presented. No presentation or tabulation of point groups is given. (The following quotation might give a wrong impression of what is meant by point group operations: 'The second class (hemihedry) of the triclinic system is the only one in the monogonal system. It cannot be called asymmetric because although it has no elements of macrosymmetry it has, like all crystallographic classes, the basic element of symmetry—the lattice.') The arrangement gives the impression of fortuitous gleaning of ripening point groups.

It can even be doubted that 'the elementary theory of the 14 Bravais lattices' is known to most class-room readers. (The word 'directions' in the following citation is apt to mislead a fresh reader: 'The fact that the monoclinic groups together with the orthorhombic ones [and $S_2^1 = P\bar{1}$] form the digonal system emphasizes the independence of all three coordinate directions in these groups.' It is further the opinion of the reviewer, that if group-theory lemmas are to be used at all the concept of a 'group' should be made clear. (The word 'group' is used in such a way as to give the impression that it is some diagram of symmetry symbols e.g. in the expression 'a mechanical superposition of one trigonal and one digonal group.') As nothing is said about what constitutes a space group either from group theory or the geometrical point of view, no discussion is, or can be, given of possible operations. The difficulties met, especially in axial classes, can therefore not easily be foreseen. It seems more desirable first to discuss the scope of one's method than to use the general strategy—aided by procedures that are not generally discussed—for example in axial classes, especially as the basic method is abandoned unnecessarily in some other classes. It is true that 'If the reader turns to any full list of space groups of symmetry he may convince himself that in the present work all 230 space groups, without exception, have been discussed.', but he would probably not have been able to tell in advance why the discussion finished where it did.

We shall turn to some minor points. For the orthorhombic space groups—but only for them—atomic coordinates and structure factors are derived, which interrupts the general line.

On p. 2 Belov duly criticizes the use of point- and space-group symbols other than the Hermann–Mauguin ones. However, he very often uses the Schoenflies symbols—often standing by themselves—for point groups and even for space groups. The modifications of the Hermann–Mauguin notation introduced in the 1952 edition of the *International Tables* were not available when Belov's paper first appeared.

For rhombohedral cells, Belov first (Fig. 13) uses the obverse relation of the centred hexagonal cell but in deriving the enantiomorphous trigonal classes it seems as though he writes in terms of a reverse setting. The use of a changed parity notation of screw axes as compared so that used in the *International Tables* adds to the confusion.

Several misprints disturb the reading. Thus axis symbols in the Hermann–Mauguin notation might appear as an index. An index in a Schoenflies symbol is more often than not wrong or appears on the line. Axis indices in Hermann–Mauguin symbols (defining screw translations) sometimes fall out (or are purposely left out for agreement with the 1935 edition of the *International Tables*). This happens consistently in the derivation of space groups isomorphous with point groups $4mm$, $4/mmm$; $6mm$, $6/mmm$. (Some of these space groups are, for an unknown reason, later rederived with indices.) This is particularly disturbing since certain axial subgroups are found by discarding the symmetry planes—but the 'babies' come out well-shaped. It also conflicts with certain reasoning presupposing rotatory tetrads and hexads.

The general impression of Belov's monograph will depend on how much emphasis is placed on the title words 'Class-room' and 'Derivation'. Many unanswered questions will occur to the critical student as the derivation is not strict. To a person acquainted with underlying hypotheses and somewhat acquainted with, say, practical use of space-group diagrams, it gives an elegant exposition, above all of the space groups of the higher classes of the different systems.

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The Major Achievements of Science.
By A. E. E. McKenzie. Two volumes, pp. xvi + [368]; [xii] + [195]. Cambridge: The University Press, 1960. Prices 30*s* and 17*s* 6*d*; school editions 20*s* and 12*s* 6*d*.

Vol. I contains an account of the historical development of the main generalizations of science and their philosophical implications. Vol. II contains a series of 91 extracts from original sources arranged in an order (only roughly chronological) suited to the chapter sequence of vol. I. The books are of great general interest to those beyond school age as well as to those below it.

It is salutary to find that crystallography does not rank among the 'major achievements' of science. The discovery of X-ray diffraction by von Laue gets five lines, as an introduction to Moseley's work on atomic numbers, and the Braggs are not mentioned.

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Die Entwicklungsgeschichte der Erde. Mit einem ABC der Geologie.
Pp. 772. Leipzig: Brockhaus Verlag, 1959. Price DM 13·80.

This book has the additional title 'Brockhaus-Taschenbuch der Geologie', but it would need a large pocket to carry it. The first section (pp. 11–540) consists of a series of articles by named experts under the general headings Allgemeine Geologie, Historische Geologie, Paläontologie, and Die Verflectung von Erd- und Lebensgeschichte. Certain articles, like Die Mineralien (Rudolf Gaedeke) and Die Entstehung der Gesteine (Rudolf Gaedeke and Carl-Dietrich Werner) will be of special interest to crystallographers, though only the NaCl structure is illustrated. The second section (pp. 543–738) is an alphabetical dictionary of geological terms. The book ends with a guide to geological literature (mostly, but not entirely, German) and an extensive index to the first section. From its level and content this book should be of great value to German students of geology and to non-Germans needing to acquire technical vocabulary, and is good value at the price.

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