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A Primer in Synchrotron Radiation: Everything You Wanted to Know about SEX (Synchrotron Emission of X-rays) but Were Afraid to Ask

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The basic properties of synchrotron radiation are derived with simple approaches, emphasizing phenomena rather than mathematical details.

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1. Introduction to SEX

Synchrotron radiation is used by tens of thousands of scientists and technologists worldwide. Almost all of them are aware, at least qualitatively, of its basic properties: angular collimation, spectral characteristics, emitted power. Yet, I believe that many of them ignore, at least in part, the simple phenomena which are responsible for such properties.

The complete treatment of synchrotron radiation can of course be found in the standard textbooks on electrodynamics, such as the classics by Jackson (1962) or by Landau & Lifchitz (1966). I seriously doubt, however, that many of the practitioners of synchrotron radiation, non-physicists in particular, have had the time to study in depth such a treatment, so as to understand its basic properties. In some cases, this leads to misconceptions.

For example, most people believe that the collimation of synchrotron radiation is, as many of its properties, a relativistic effect. Strictly speaking, however, this is not true: some degree of collimation is also present for very classic wave phenomena such as the emission of sound from a moving source. On the other hand, the collimation becomes extreme for synchrotron radiation because the speed of the source is close to c, and in this sense it is a relativistic phenomenon.

Many texts of synchrotron radiation, including one of my own (Margaritondo, 1988), did not contribute sufficiently to a better understanding of synchrotron radiation properties, since they either presented complicated formalism or simply the end results, with little or no explanation of the underlying physics.

I therefore believe that it might be interesting for users of synchrotron radiation and curious bystanders to have a simple description of the physical nature of the most important properties of synchrotron radiation.* My objective is to explain in simple terms the following properties: angular collimation, the spectrum of an undulator, the total power emitted by a bending magnet, the time duration of one of the pulses of the bending magnet, and the spectral distribution of such pulses. In order to simplify the understanding of the derivations, the symbols are summarized in Table 1 and the most important results in Table 2.

Table 1
Symbols.

	In the source reference frame, F_S	In the laboratory reference frame, F_L
Axis parallel to the velocity of the electron	x_S	x_L
Axis perpendicular to the velocity of the electron	y_S	y_L
Electron velocity		v
Electron energy		E
Electron *rest* mass	m_0	m_0
Angular frequency and photon frequency	ω_S	ω_L
Photon energy	$E_S = \hbar\omega_S$	$E_L = \hbar\omega_L$
Photon momentum components	$p_{Sx}; p_{Sy}$	$p_{Lx}; p_{Ly}$
Direction of light emission	θ_S	θ_L
Magnetic field strength		B_L
Bending radius		R_L
Total emitted power		P
Undulator period		λ_L
Undulator K parameter		K
Undulator number of periods	N_u	N_u
Bending magnet pulse duration		δt_L
Critical frequency (bending magents)		ω_{cL}
Critical photon energy (bending magnets)		$\hbar\omega_{cL}$

* Readers interested in expanding their knowledge of synchrotron radiation can find a number of excellent texts, in particular Bachrach (1992) and Koch (1983), and for undulators, Elleaume (1992) and Brown (1992).

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Table 2
Main results.

$\tan\theta_L \simeq \sin\theta_S/\gamma(\cos\theta_S+1)$	(16)
$\theta_L <\sim 1/\gamma$	
$\omega_L = eB_L/\gamma m_0$	(17)
$R_L \simeq (m_0 c/e)(\gamma/B_L)$	(18)
$R \propto (E/B_L)$	(19)
$P \propto i\gamma^4/R_L \propto iE^4/R_L$	(20)
$K = eB_L\lambda_L/2\pi c m_0$	(24)
$\hbar\omega_L \simeq (\hbar 4\pi c\gamma^2/\lambda_L)[1/(1+\frac{1}{2}K^2+\theta_L{}^2\gamma^2)]$	(25)
$\Delta\hbar\omega_L/\omega_L = 1/N_u$	(26)
$\delta\theta_L \simeq (1+K^2/2)^{1/2}/\gamma N_u{}^{1/2}$	(27)
$\hbar\omega_{cL} \simeq 2\hbar c\gamma^3/R_L$	(29)
$\hbar\omega_{cL} \simeq \dfrac{2\hbar\gamma^2 eB_L}{m_0} = \left(\dfrac{2\hbar e}{m_0{}^3c^4}\right)(E^2B_L)$	(30)

I must emphasize that no new physics will be derived, but only (hopefully) simpler, although approximate, versions of previous treatments. I must also emphasize that several of the derivations are not, or not entirely, new. For the sake of completeness and clarity, I decided nevertheless to include all of them. In most cases, acknowledging their origins is not easy, since the way of thinking has been in the background of synchrotron radiation for a long time but has seldom been formally published.

2. The Doppler effect

Since many properties of synchrotron radiation derive from the relativistic Doppler effect, we would like to remind the reader how the basic equations of this effect are derived.

Consider a source of light which moves at speed v along the x direction. For simplicity, we will limit our analysis to a plane, defined by the x direction and by the y direction, as shown in Fig. 1. We will consider all phenomena in two different reference frames: F_S, the frame moving with the source, and F_L, the laboratory or observation frame. Seen from F_S, the frame F_L moves along the x_S axis with the velocity $-v$.

We will hereafter label all quantities measured in reference frame F_S with the subscript S, and all those measured in F_L with the subscript L. The standard Lorentz transformations are, of course

$$x_L = \gamma(x_S + vt_S), \quad (1)$$

$$y_L = y_S, \quad (2)$$

$$t_L = (t_S + vx_S/c^2), \quad (3)$$

where

$$\gamma = (1-\beta^2)^{-1/2}, \quad (4)$$

$$\beta = v/c. \quad (5)$$

Note that if the moving source is an electron in a storage ring, γ is also proportional to the electron's (relativistic) energy,

$$\gamma = E/m_0c^2, \quad (6)$$

where m_0 is the rest mass.

The simplest way to derive the Dopper effect laws is to consider the Lorentz transformations for momentum and energy,

$$p_{Lx} = \gamma(p_{Sx} + vE_S/c^2) \quad (1a)$$

$$p_{Ly} = p_{Sy} \quad (2a)$$

$$E_L = \gamma(E_S + vp_{Sx}). \quad (3a)$$

Consider now a photon emitted by the source along the x axis, for which $E_S = \hbar\omega_S$, $p_{Sx} = \hbar\omega_S/c$, $p_{Sy} = 0$, and $E_L = \hbar\omega_L$. Equation (3a) gives immediately $\hbar\omega_L = \gamma(\hbar\omega_S + v\hbar\omega_S/c) = \hbar\omega_S\gamma(1+\beta)$, and therefore

$$\omega_L = \omega_S[(1+\beta)/(1-\beta)]^{1/2}, \quad (7)$$

which is the best known form of the relativistic Doppler effect. Note that for relativistic speeds, $v \to c$ and therefore $\beta \to 1$

$$\left(\frac{1+\beta}{1-\beta}\right)^{1/2} = \frac{1+\beta}{(1-\beta^2)^{1/2}} = \gamma(1+\beta) \simeq 2\gamma,$$

and

$$\omega_L \simeq 2\gamma\omega_S. \quad (8)$$

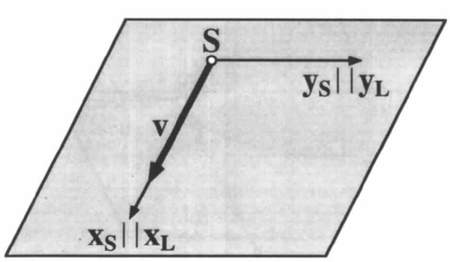

Figure 1
Coordinate systems: x_S and y_S in the source reference frame F_S, and x_L and y_L in the laboratory reference frame F_L. The source S is moving at the speed v along the x axis in the laboratory frame.

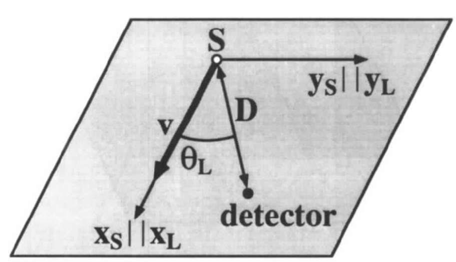

Figure 2
Geometry of the Doppler effect in the xy plane, with the detector at the angle θ_L with respect to the source.

We must now generalize our derivation assuming that the detector is not along the x direction of the velocity of the source, but at an angle θ_L as shown in Fig. 2. Since we want to express the results in terms of θ_L, we will take the inverse transformation of (3*a*):

$$E_S = \gamma(E_L - vp_{Lx}), \tag{3b}$$

and, since $p_{Lx} = (\hbar\omega_L/c)\cos\theta_L$, then $\hbar\omega_S = \gamma[\hbar\omega_L - v(\hbar\omega_L/c)\cos\theta_L] = \hbar\omega_L\gamma(1-\beta\cos\theta_L)$, giving

$$\omega_L = \omega_S[1/\gamma(1-\beta\cos\theta_L)], \tag{9}$$

which indeed reduces to (7) for $\theta_L = 0$, and becomes $\omega_L = \gamma\omega_S$ for $\theta_L = \pi/2$, the transverse relativistic Doppler effect.

3. Angular collimation

One phenomenon is common to all types of synchrotron radiation sources: angular collimation. This is also a common phenomenon to all wave sources in motion with respect to the detector. Consider, for example, a collimated sound wave emitted at an angle θ_S with respect to the motion of its source (see Fig. 3). We can write

$$\theta_S = \tan^{-1}(u_{Sy}/u_{Sx}), \tag{10}$$

where $u_{Sy} = u_S\sin\theta_S$ and $u_{Sx} = u_S\cos\theta_S$ are the components of the wave's velocity in the source frame F_S (and u_S is of course its magnitude).

The angle θ_L of the wave's (detection) direction in the laboratory frame F_L is given by

$$\theta_L = \tan^{-1}(u_{Ly}/u_{Lx}), \tag{11}$$

where u_{Ly} and u_{Lx} are the components of the velocity of the sound wave in F_L. These can be derived from the corresponding components u_{Sy} and u_{Sx}. We will use in this case the non-relativistic Galilean transformation of velocity, $u_{Ly} = u_{Sy} = u_S\sin\theta_S$; $u_{Lx} = u_{Sx} + v = u_S\cos\theta_S + v$, obtaining

$$\theta_L = \tan^{-1}\left(\frac{u_S\sin\theta_S}{u_S\cos\theta_S + v}\right) = \tan^{-1}\left(\frac{\sin\theta_S}{\cos\theta_S + \dfrac{v}{u_S}}\right). \tag{12}$$

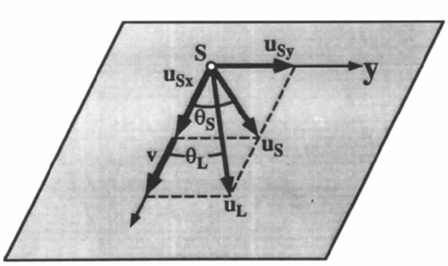

Figure 3
Collimation caused by the motion of the source for a classical wave: because of the Galilean transformation of the velocity, the direction of propagation of the sound changes from θ_S to θ_L.

The angle θ_L is then smaller than θ_S, and the reduction is larger if the speed of the source is close to that of the wave. That is why, in the case of light waves like synchrotron radiation, the emission is collimated if the source, the electron, moves at relativistic speed.

We now consider synchrotron radiation: an electron in non-relativistic motion with instantaneous centripetal acceleration emits waves with an intensity pattern $\simeq (1-\sin^2\theta\cos^2\varphi)$ where θ and φ are the angles defining the light's emission direction with respect to the velocity of the electrons and to the plane of their orbit, as shown in Fig. 4. In the plane of the orbit ($\varphi = 0$), the pattern becomes $\simeq \cos^2\theta$. This pattern corresponds to emission over a broad range of angles; for example, the intensity is decreased only by a factor of two on going from $\theta = 0$ to $\pi/4$.

In the relativistic case, the pattern $\simeq \cos^2\theta_S$ is the one seen in the frame F_S. Note that there is emission in F_S since the velocity of the electron is zero, but *its (centripetal) acceleration is not*. In fact, the *inertial* frame F_S is the one that instantaneously coincides with the position of the electron and has the same velocity, not the (non-inertial) frame that follows the electron. The pattern in F_L is much more collimated: the emitted intensity is concentrated to a small angular range close to the tangential direction. This is the same type of phenomenon as for the non-relativistic case. However, the Lorentz transform properties enhance its importance.

Suppose we have a light beam moving along the θ_S direction in F_S: what is its direction θ_L in F_L? We can simply use the energy-momentum Lorentz transformations, equations (1*a*)–(3*a*), remembering that $p_{Sx} = (\hbar\omega_S/c)\cos\theta_S$, $p_{Sy} = (\hbar\omega_S/c)\sin\theta_S$, $p_{Lx} = (\hbar\omega_L/c)\cos\theta_L$, $p_{Ly} = (\hbar\omega_L/c) \times \sin\theta_L$, therefore

$$\tan\theta_L = \left(\frac{p_{Ly}}{p_{Lx}}\right) = \left[\frac{p_{Sy}}{\gamma(p_{Sx} + vE_S/c^2)}\right], \tag{13}$$

which, using (1*a*) and (2*a*) becomes

$$\tan\theta_L = \left\{\frac{(\hbar\omega_S/c)\sin\theta_S}{\gamma[(\hbar\omega_S/c)\cos\theta_S + v\hbar\omega_S/c^2]}\right\}, \tag{14}$$

and therefore

$$\tan\theta_L = \sin\theta_S/\gamma(\cos\theta_S + \beta). \tag{15}$$

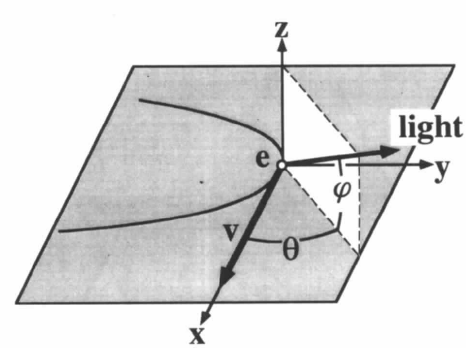

Figure 4
Definition of the polar coordinate system used to analyze the spatial distribution of the emitted radiation.

For relativistic electrons, $\beta \simeq 1$, and

$$\tan\theta_L \simeq \sin\theta_S/\gamma(\cos\theta_S + 1). \quad (16)$$

This equation explains the extreme collimation of the emitted light in F_L. For example, the half-intensity direction $\theta_S = \pi/4$ in F_S corresponds in F_L to an angle $\theta_L \simeq \tan^{-1}[0.41(1/\gamma)]$, which is extremely small for the relativistic case $\gamma \gg 1$.

The value $1/\gamma$ essentially determines the angular width of the beam: one can, in fact, demonstrate that the intensity's angular distribution pattern in F_L is approximately proportional to the factor $[1-(\gamma\theta_L)^2]^2$, which becomes zero for $\theta_L = 1/\gamma$.

4. Total emitted power from bending magnets

We will now identify the factors that determine the total power emitted by a relativistic electron in circular motion under the influence of a constant magnetic field of strength B_L (in the laboratory frame, F_L). The motion created by this field is indeed circular (if the vertical component of the velocity is zero) both in the classical case and in the relativistic case.

In the classical case, the frequency of the cyclotron motion is given by $\omega_L = eB_L/m_0$. In the relativistic case, we must replace m_0 with γm_0, obtaining again a circular cyclotron motion of frequency

$$\omega_L = eB_L/\gamma m_0 \quad (17)$$

(seen from F_L). Since ω_L is also given by $v/R_L \simeq c/R_L$ (R_L = radius of curvature), we have

$$R_L \simeq (m_0c/e)(\gamma/B_L). \quad (18)$$

Using (6), this approximate result can also be written as

$$R \propto (E/B_L), \quad (19)$$

where E is the energy of the electron.

In classical electrodynamics (Jackson, 1962; Landau & Lifchitz, 1966), an electrically charged particle circulating in a circular orbit with speed v would emit a total power P proportional to the square of the acceleration, $P \propto a^2$. On the other hand, the acceleration is proportional to $v\omega_L$, therefore it is also proportional to vB and to pB_L, where p is the particle's momentum magnitude. Therefore, the total radiated power is proportional to $(pB_L)^2$.

This is the important result, since it is also valid for a relativistic particle (Landau & Lifchitz, 1966). Only, for a relativistic case, the momentum magnitude is $p = \gamma m_0 v \simeq \gamma m_0 c \propto \gamma$, thus the total radiated power is proportional to $\gamma^2 B_L{}^2$. Using (16), this is also proportional to $\gamma^4/R_L{}^2$.

This result is valid for each of the electrons circulating in a storage ring. The total power emitted by the ring is obtained by multiplication by the number of circulating electrons, N. On the other hand, the circulating current i is proportional to N/τ_L, where $\tau_L \simeq 2\pi R_L/c$ is the rotation time of the electrons around the ring. Therefore, N is proportional to iR_L, and the total emitted power

$$P \propto i\gamma^4/R_L \propto iE^4/R_L. \quad (20)$$

We have therefore found all of the relevant factors in P, and in particular the rather dramatic dependence on the fourth power of the electrons' energy. Note that this dependence derives from two relativistic effects: the energy dependence of the relativistic momentum, and the energy dependence of the relativistic cyclotron frequency.

5. Emission spectrum: undulators

The simplest case of spectral distribution of synchroton radiation is that of the linear undulator, *i.e.* a periodic series of magnets causing small undulations in the otherwise rectilinear trajectory of the electron beam, whose emissions combine together to give a narrow spectral line of extreme brightness (Margaritondo, 1988). This distribution is entirely determined by the Lorentz contraction and by the Doppler effect.

We will first simplify the analysis by neglecting the angular deviations caused by the magnetic field-induced undulation, and assume that the emitted light is detected along the axis of the undulator ($\theta_S = \theta_L = 0$). An undulator of period λ_L causes the emission of synchrotron radiation with the 'corresponding' wavelength. 'Corresponding' means that in the reference frame F_S the wavelength is λ_L corrected for the Lorentz contraction, λ_L/γ, as schematically illustrated in Fig. 5. The photon energy corresponding to this wavelength is $\hbar\omega_S = \hbar 2\pi c\gamma/\lambda_L$.

The photon energy seen from F_L is, according to (8), Doppler-shifted by the (approximate) factor 2γ,

$$\hbar\omega_L = \hbar 4\pi c\gamma^2/\lambda_L. \quad (21)$$

Therefore, the undulator's (first harmonic) photon energy is simply the result of the Lorentz contraction and of the Doppler shift for collinear motion.

We will now slightly complicate the analysis by assuming that the light is detected at an angle θ_L from the axis. The

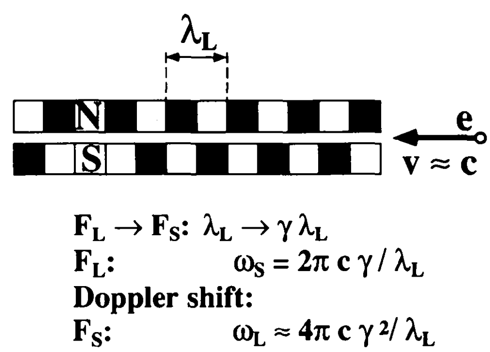

Figure 5
A zero-order derivation of the first-harmonic photon frequency ω_L for a linear undulator. The frequency is determined by the period of the undulator λ_L corrected for the Lorentz contraction and for the Doppler shift.

emission of the undulator is confined to angles smaller than $\sim 1/\gamma$, thus θ_L must be very small.

The Doppler shift factor is given by (9), and can be approximated as

$$1/\gamma(1-\beta\cos\theta_L) \simeq 1/\gamma\left[1-\beta\left(1-\frac{\theta_L{}^2}{2}\right)\right]$$
$$= 1/\gamma(1-\beta)\left[1+\frac{\beta\theta_L{}^2}{2(1-\beta)}\right].$$

On the other hand

$$\gamma(1-\beta) = \frac{(1-\beta)}{(1-\beta)^{1/2}} = \frac{(1-\beta)^{1/2}}{(1+\beta)^{1/2}} = \frac{(1-\beta^2)^{1/2}}{(1+\beta)} \simeq \frac{1}{2\gamma},$$

thus the Doppler factor is

$$\simeq 2\gamma\left[1+\frac{\beta\theta_L{}^2}{2(1-\beta)}\right]^{-1} = 2\gamma\left[1+\frac{\beta\theta_L{}^2(1+\beta)}{2(1-\beta^2)}\right]^{-1}$$
$$\simeq 2\gamma/(1+\theta_L{}^2\gamma^2)$$

The emitted photon energy along θ_L is, therefore,

$$\hbar\omega_L \simeq (\hbar 4\pi c\gamma^2/\lambda_L)\left[\frac{1}{(1+\theta_L{}^2\gamma^2)}\right], \tag{22}$$

which correctly describes the angular dependence of the undulator radiation.

Next, we must further refine the analysis by taking into account the undulations caused by the magnetic field. Once again, their effects can be understood in terms of Doppler shifting. We could describe them as corrections of the θ_L angle.

An alternative way is to consider that the undulations dynamically reduce the x component of the velocity of the electron with respect to its speed, v. Consider the emission at $\theta_L = 0$; the Doppler factor is $\simeq 2\gamma = 2/(1-\beta^2)^{1/2}$; because of the undulations, however, $\beta = v/c$ must be replaced by the ratio $v_\parallel/c$, where $v_\parallel$ is the component of the electron's velocity along the undulator axis. In turn, $v_\parallel = v(1-\cos\zeta) \simeq v(1-\zeta^2/2)$, where ζ is the (small) deviation angle with respect to the undulator axis. This angle changes with time, but we will consider an average value, $\langle\zeta\rangle$.

The angle ζ is related to the rotation angle caused by the Lorentz force of the undulator's magnetic field. According to (17), the angular velocity in the relativistic case is $eB_L/\gamma m_0$; we will then assume that the average $\langle\zeta\rangle$ is of the order of magnitude of the rotation angle caused by this angular velocity during the time $(\lambda_L/4v) \simeq (\lambda_L/4c)$, the time necessary for the electron to travel a quarter period, $\lambda_L/4$, along the undulator (see Fig. 6). Therefore, $\langle\zeta\rangle \propto (eB_L/\gamma m_0)(\lambda_L/4c)$, and on the average $v_\parallel \simeq v[1-(AeB\lambda_L/4c\gamma m_0)^2/2]$, where A is a constant. The Doppler factor then becomes:

$$\simeq 2\left[1-\left(\frac{v_\parallel}{c}\right)^2\right]^{-1/2} = 2\left\{1-\left[\beta\left(1-\frac{\langle\zeta\rangle^2}{2}\right)\right]^2\right\}^{-1/2}$$
$$\simeq 2\left\{1-\beta^2\left[1-\tfrac{1}{2}\left(\frac{AeB_L\lambda_L}{4c\gamma m_0}\right)^2\right]^2\right\}^{-1/2}.$$

Calling $K = (AeB\lambda_L/4cm_0)$, this factor is approximately

$$\simeq 2/\{1-\beta^2[1-(K/\gamma)^2]\}^{1/2} \simeq 2\gamma/(1+\beta^2K^2)^{1/2}$$
$$\simeq 2\gamma/(1+\tfrac{1}{2}K^2),$$

and we can write

$$\hbar\omega_L \simeq (\hbar 4\pi c\gamma^2/\lambda_L)[1/(1+\tfrac{1}{2}K^2)]. \tag{23}$$

This equation does contain all of the important factors that determine the effects of the undulations. A less approximate derivation would specify the value of the constant A in the parameter $K = (AeB_L\lambda_L/4cm_0)$, giving

$$K = eB_L\lambda_L/2\pi cm_0, \tag{24}$$

the K parameter of the undulator, which corresponds to the deviation angle with respect to the axis, measured in units of $(1/\gamma)$.

We can now combine the two corrections of (22) and (23), neglecting higher-order terms; the result is the well known first-harmonic undulator equation:

$$\hbar\omega_L \simeq (\hbar 4\pi c\gamma^2/\lambda_L)[1/(1+\tfrac{1}{2}K^2+\theta_L{}^2\gamma^2)]. \tag{25}$$

It should be noted that the bandwidth of the emitted photon energy also depends on the number of periods, N_u. This is primarily the usual effect of coherent combination of waves that one finds, for example, in diffracton gratings or in the X-ray scattering of crystals. We can, therefore, use the general result

$$\Delta\hbar\omega_L/\omega_L = 1/N_u. \tag{26}$$

Note, however, that the bandwidth cannot be decreased by increasing the number of periods beyond certain limits, for example those caused by the energy spread of the electron beam.

If we differentiate the logarithm of (25) with respect to the variable $\theta_L{}^2$, we can easily obtain (for $\theta_L \simeq 0$): $\delta\theta_L \simeq (1/\gamma)(1+K^2/2)^{1/2}(\Delta\hbar\omega_L/\omega_L)^{1/2}$; using (26), this expression becomes

$$\delta\theta_L \simeq [(1+K^2/2)^{1/2}/N_u{}^{1/2}](1/\gamma), \tag{27}$$

which implies that the angular spread of the undulator's emission over its narrow bandwidth $\Delta\hbar\omega_L$ is substantially smaller than the $(1/\gamma)$ value derived from (16). This extreme angular collimation and the corresponding very

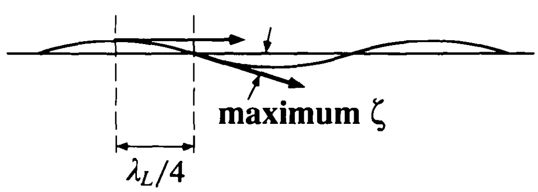

Figure 6
Rough estimate of the maximum value of the angle of detection ζ for an undulator, which corresponds to the deflection caused by the Lorentz force during the motion along $\lambda_L/4$.

high brightness is one of the most remarkable properties of undulators.

An undulator emits not only the first harmonic whose properties are described by equations (25)–(27), but also higher order harmonics of photon energy $n\hbar\omega_L$, whose properties can be easily derived by modifying the same equations, for example, by replacing N_u with nN_u in (26) and (27).

6. Time structure of the bending magnet emission

The narrow angular width of the photon beam emitted by a bending magnet makes it similar to a very collimated searchlight. We will now find the duration of the pulse of light seen by a point detector.

Assume, as shown in Fig. 7, that the detector is in the plane of the storage ring at a distance D_L from the orbit of the electrons (D_L is measured in the tangential direction). Assume that the emission of the detected pulse starts at time $t_L = 0$ (in the frame F_L); its emission will end at the time Δ_L/v, where Δ_L is the distance along which the electron travels between the beginning and the end of the emissions of the detected pulse.

Consider now the detection times: the light emitted at $t_L = 0$ is detected at the time $t_L = D_L/c$; the light emitted at $t_L = \Delta_L/v$ is detected at $t_L = \Delta_L/v + (D_L - \Delta_L)/c$. Note that the distance the light travels is no longer D_L, but $(D_L - \Delta_L)$. The difference between detection times is

$$\delta t_L = \left(\frac{\Delta_L}{v} + \frac{D_L - \Delta_L}{c}\right) - \frac{D_L}{c} = \frac{\Delta_L}{v} - \frac{\Delta_L}{c};$$

on the other hand, the change in the tangential direction angle, Δ_L/R_L, must of course equal the angular aperture $1/\gamma$ of the 'searchlight', thus $\Delta_L \simeq R_L/\gamma$, and

$$\delta t_L \simeq \left(\frac{R_L}{\gamma}\right)\frac{1-\beta}{c\beta} = \left(\frac{R_L}{\gamma}\right)\frac{1-\beta^2}{c\beta(1+\beta)} = \left(\frac{R_L}{\gamma^3 c}\right)\frac{1}{\beta(1+\beta)}.$$

For $\beta \simeq 1$,

$$\delta t_L \simeq R_L/2c\gamma^3, \tag{28}$$

an expression which contains the correct factors of the pulse duration (Jackson, 1962; Landau & Lifchitz, 1966; Margaritondo, 1988).

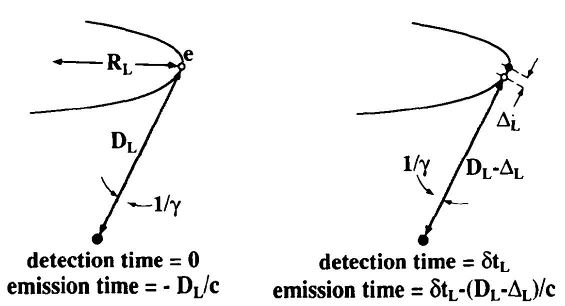

Figure 7
Estimate of the duration time of a pulse of synchrotron radiation, *i.e.* of the time distance δt_L between the detection of the leading edge of the angular distribution and the detection of the end edge.

7. Spectral properties of the bending magnet emission

The time structure of the emitted bending magnet radiation also influences its spectral distribution. The common way to explain this point is to say that a time pulse of width $\delta t_L \simeq (R_L/2c\gamma^3)$ has Fourier components up to a frequency of the order of $\omega_{cL} \simeq 1/\delta t_L \simeq (2c\gamma^3/R_L)$. This would explain the order of magnitude of the 'critical photon energy' (Jackson, 1962; Landau & Lifchitz, 1966; Margaritondo, 1988) for bending magnet radiation,

$$\hbar\omega_{cL} \simeq 2\hbar c\gamma^3/R_L, \tag{29}$$

which using (18) can also be written

$$\hbar\omega_{cL} \simeq 2\hbar\gamma^2 eB_L/m_0 = (2\hbar e/m_0{}^3c^4)(E^2B_L). \tag{30}$$

This explanation, however, misses in my opinion one important point. The critical photon energy is not, or not simply, as many people believe, the 'cut-off photon energy' for the spectral emission of the synchrotron. The critical photon energy marks the spectral point for which one half of the total power is irradiated at lower photon energies, and one half at higher (Margaritondo, 1988). In a sense, therefore, it should be thought of as the 'central' point of the distribution rather than as the 'cut-off', even if the 'cut-off' energy is still related to $\hbar\omega_{cL}$. The perception that $\hbar\omega_{cL}$ is the 'cut-off' point is probably strengthened by the conventional way of plotting the spectral distribution, using a log–log scale, which emphasizes the low photon energy part of the distribution.

The idea of a 'central' point in the distribution can be understood as follows. First of all, we must note again that, strictly speaking, the reference frame F_S is not the frame moving with the source, but the frame in linear motion whose velocity instantaneously coincides with the (tangential) velocity of the source. Thus, the electron is accelerated in such a frame, and emits radiation.

In the classical case, the cyclotron motion of (angular) frequency $\omega_L = eB_L/m_0$ would cause the emission of photons whose spectrum is centered at the same frequency. In the relativistic case, the transverse Lorentz force of strength evB_L seen in F_S becomes an *electrostatic* force due to a transverse *electric* field of strength γvB_L. This force must correspond to m_0 times the acceleration v^2/R_L, thus $m_o v^2/R_L = \gamma veB_L$ and

$$v/R_L = \gamma eB_L/m_0. \tag{31}$$

This would give a radiation emission centered at the frequency (seen from F_S) of $v/R_L = \gamma eB_L/m_0$.

After Doppler shifting, the frequency seen in F_L is:

$$\omega_{cL} \simeq (2\gamma)(\gamma eB_L/m_0) = \text{constant} \times E^2B_L, \tag{32}$$

where E is the energy of the electrons and the constant

$= (2e/m_0{}^3c^4)$. Equation (32) gives then a 'central' photon energy in the synchrotron radiation emission spectrum, $\hbar\omega_{cL}$, coincident with the 'critical' photon energy of (30).

On the other hand, the short duration of the photon pulse affects the bandwidth of the distribution, due to the uncertainty principle (*i.e.* again, to the Fourier theorem). The total bandwidth is of the order of $\simeq \hbar(1/\delta t_L) \simeq (2\hbar c\gamma^3/R_L)$, *i.e.* of the same order as the magnitude of the 'central' photon energy.

One can try to guess roughly the lineshape of the distribution by assuming that the broadening produces a Gaussian lineshape, of width $(2\hbar c\gamma^3/R_L) = (2\hbar e/m_0{}^3c^4) \times(E^2B_L)$ and centred at $\hbar\omega_{cL} \simeq (2\hbar e/m_0{}^3c^4)(E^2B_L)$. This lineshape is shown in Fig. 8, both as a linear–linear and as a log–log plot. One can see that it does roughly reproduce the well known characteristics of the bending magnet's spectral distribution, except for a displacement of the critical photon energy which is caused by the approximations used in its derivation.

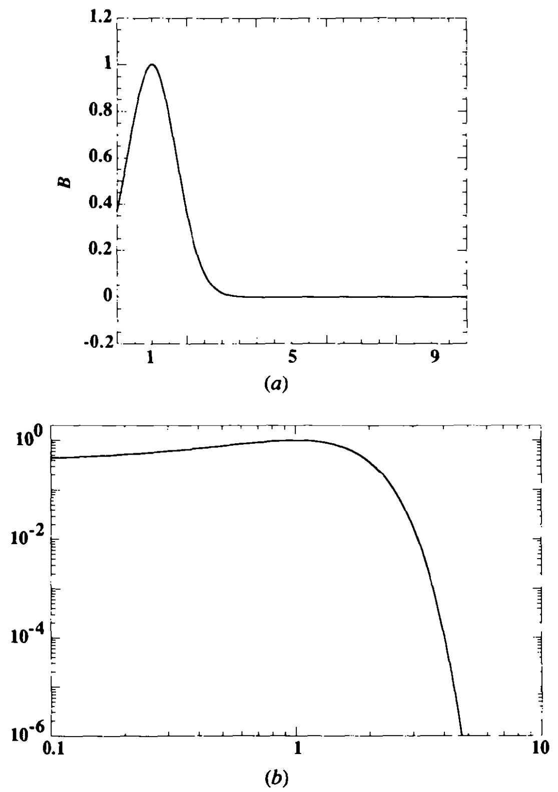

Figure 8
Rough estimate of the spectral distribution of synchrotron radiation from bending magnets: (*a*) linear–linear plot of intensity *versus* photon energy normalized to the critical value; (*b*) log–log plot of the same function.

From such a derivation, we have learned that the spectral characteristics of the bending magnet radiation are the result of two factors: the Doppler shifting of the emission, and the broadening due to the short duration of the pulse.

The connection between the 'searchlight' character of bending magnet radiation and its spectral broadening enables us to understand the difference between bending magnets and undulators. In the latter, the undulations are so small that the detector is continuously illuminated by the 'searchlight'. The emission time structure no longer consists of the pulses defined by (28), but is the random superposition of wavetrains of N_u wavelengths each. The broadening due to the 'searchlight' effect does not take place and the radiation is concentrated in the narrow bandwidth defined by (26).

The undulator condition of small angular undulations is satisfied if the value of K is sufficiently small, since $K\gamma$ corresponds to the angular deviation of the undulator. For large K's, one instead reaches the wiggler limit with large spectral bandwidth.

8. Conclusions

Synchrotron radiation is simple: enjoy it!

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