J. Synchrotron Rad. (1996). 3, 268–271

A Technique for Generating Potent Positron Beams

C. A. Kapetanakos†

Institute of Plasma Physics, University of Crete, Greece, and LET Corporation, Washington, DC, USA

(Received 23 May 1996; accepted 31 July 1996)

This paper reports on a novel scheme that has the potential to generate intense positron beams. It is based on the modified betatron accelerator, a compact high-current device that has been developed in the last few years. Briefly, the proposed accelerator consists of two modified betatron accelerators that are stacked together and share the same core. The electrons are accelerated in the upper torus during the first half of the flux waveform when the time rate of the magnetic flux (φ) is positive. After completion of the acceleration, the electrons are extracted and guided to a high-Z target producing a positron beam that is accelerated in the lower torus during the second half of the waveform when $d\varphi/dt$ is negative.

Keywords: positron beams; accelerators; modified betatron; ZARA-tron.

1. Introduction

Existing and next-generation colliders, both linear and circular (Wurtel, 1994; Stiening, 1987; Barletta, 1993), require high-luminosity (high current, low emittance) positron beams to provide the desired event rate. In addition, intense positron beams are required for the generation of synchrotron radiation for a host of applications (Hasnain, Helliwell & Kamitsubo, 1994) such as X-ray lithography, diffraction, spectroscopy and imaging. Finally, the availability of potent positron beams will provide a new tool that may lead to applications that have not yet been discovered.

The trapping of ions in the potential well of electron beams in storage rings (Barton, 1985) limits the electron-beam lifetime, enhances its emittance and reduces its reliability. These shortcomings are due to enhanced scattering between electrons and ions, to various streaming instabilities, and to the non-linear nature of the ionic potential well. Experiments with DCI at Orsay and with SSRL at Palo Alto (Barton, 1985) have demonstrated that these shortcomings can be avoided by swapping the electrons with positrons.

This paper reports on a novel scheme that has the potential to generate potent positron beams. It is based on the modified betatron accelerator (Sprangle & Kapetanakos, 1978; Kapetanakos *et al.*, 1993), a compact high-current recirculating accelerator that has been developed in the last few years. The improved current-carrying capability of the modified betatron accelerator is due to the addition of a toroidal magnetic field, B_θ, and a strong focusing twisted quadrupole (Gluckstern, 1979; Roberson, Mondelli & Chernin, 1983) to the vertical B_z field of the conventional betatron (Kerst, Adams, Koch & Robirson, 1950).

† Currently at LET Corporation.

In the betatron the electron beam is confined in the transverse plane by the gradient of the B_z field. On the minor axis of the devices $(\partial B_z/\partial r)_{r_0,0} = -nB_{z0}/r_0$, where n is the external field index that has a value between 0 and 1, B_{z0} is the vertical magnetic field on the minor axis, and r_0 is the major radius of the device. For $n = 0.5$, $r_0 = 100$ cm and $B_{z0} = 30$ G (at injection), $(\partial B_z/\partial r)_{r_0,0} = -0.15$ G cm^{-1}.

In the modified betatron accelerator the rotating quadrupole produces on the minor axis a field with maximum gradient $(\partial B_z/\partial r)^{max}_{r_0,0} = 4\alpha^2 B_0 \rho_0 K_2'(2\alpha\rho_0)$, where $\alpha = 2\pi/L$, $B_0 = 2\mu_0 I_{st}/L$, L is the period, I_{st} is the current, ρ_0 is the radius of the windings, and K_2' is the derivative of the modified Bessel function. For the Naval Research Laboratory (NRL) modified betatron accelerator, $\alpha = 0.03$ cm^{-1}, $L = 2.0933$ m, $I_{st} = 30$ kA and the ratio $(\partial B_z/\partial r)^{max}_{r_0,0}/(\partial B_z/\partial r)_{r_0,0}$ is 137.5. The improved current-carrying capability of the modified betatron accelerator is a direct consequence of this high ratio.

The research program to develop the modified betatron accelerator at the NRL lasted several years and furnished valuable information on the various critical physics issues of the concept (Kapetanakos *et al.*, 1993). During the last phase of its operation, the trapped electron current in the device routinely exceeded 1 kA and the peak energy exceeded 20 MeV. All the electron rings in the NRL device were formed by an injected electron beam, typical energies being in the range 0.5–0.6 MeV. A smaller modified betatron accelerator at the University of California, Irvine (Ishizuka, Prohasko, Fisher & Rostoker, 1988), also obtained interesting results. However, the majority of electron rings in the Irvine device were formed by runaway electrons and not by an injected beam.

2. Background

To obtain some insight into the physics of the proposed concept, consider an electron and a positron that gyrate in the same direction, as shown in Fig. 1. The negatively charged electron requires a positive B_z to spin counterclockwise and a positive $d\varphi/dt$ in order to gain energy by the changing magnetic flux, φ. In contrast, both B_z and $d\varphi/dt$ are negative for a positron. Fig. 2(*a*) shows the current waveform that powers the flux coil in a betatron. From 0 to π, $d\varphi/dt$ is positive, while between π and 2π, $d\varphi/dt$ is negative. The solid line in Fig. 2(*b*) shows the corresponding vertical field waveform. By superimposing a static positive-bias magnetic field that is equal to the peak amplitude of the sinusoidal waveform, the combined field will be positive at all times, as shown by the dashed curve in Fig. 2(*b*). Similarly, a static negative-bias magnetic field will make the total vertical field negative, as shown by the dotted curve in Fig. 2(*b*). Therefore, the intervals from 0 to π, 2π to 3π, 4π to 5π . . . are suitable for confining and accelerating electrons, while the intervals from π to 2π, 3π to 4π, 5π to 6π . . . are suitable for confining and accelerating positrons.

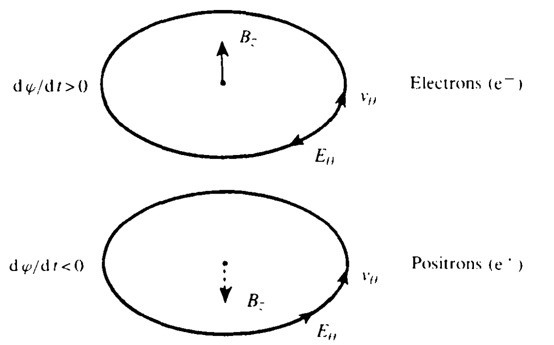

Figure 1
Electron and positron orbits in a uniform time-varying magnetic field. E_0 is the accelerating inductive field and φ is the magnetic flux.

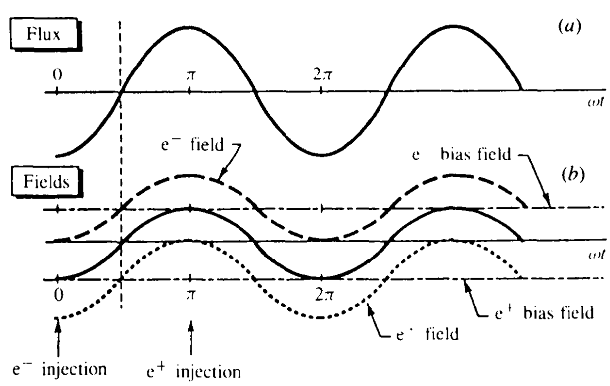

Figure 2
The solid curves show (*a*) the flux, and (*b*) vertical magnetic field waveforms in a betatron. The dashed curve in (*b*) shows the field waveform when a static positive magnetic field is superimposed on the original waveform. This combination is used in the upper modified betatron accelerator of Fig. 3. The dotted curve results from the original and a negative-bias field and is used in the lower modified betatron accelerator of Fig. 3.

3. Description of the device

Fig. 3 shows a device that has the desired features. It consists of two stacked modified betatron accelerators that share the same core. The upper modified betatron accelerator has a positive bias field while the bias field in the lower modified betatron accelerator is negative. The electrons are injected at $\omega t \simeq 0$ and reach their peak energy at $\omega t \cong \pi$. Just before the peak energy is reached, both the external field index and the strong focusing field are brought to zero and a rapidly rising toroidal electric field is applied at a gap in the torus. The electron beam drifts in the vertical direction with a speed that is several centimetres per revolution. After a few revolutions the beam enters the extractor, a short helical pipe. Details about this extraction technique may be found in Kapetanakos, Drakakis, Xenidi & Karabourniotis (1995). It is apparent from Fig. 3 that the proposed device has a zygomorphic shape and thus an appropriate name for it is Zygomorphic Advanced Recirculating Accelerator, or for short ZARA-tron.

The electron beam propagates in the helical extractor and strikes a high-Z target. Positrons are produced *via* the electromagnetic cascade that is initiated by the electron beam inside the target. In general, the positrons are accompanied by a large number of electrons. The majority of positrons are produced at a large angle with respect to the direction of the incident electron beam. A z-pinch confines and guides the positrons to the lower torus as shown in Fig. 4. The electrons have the wrong polarity and thus are not confined by the magnetic field of the z-pinch. As a result of the coaxial character of the z-pinch, its magnetic field is localized inside the helical pipe and therefore will not disturb the fields of the modified betatron accelerators.

4. Transport of positrons

The equations of motion that describe the orbit of positrons in the helical pipe of Fig. 4, in the presence of various external magnetic fields, have been solved analytically. However, the expressions are very complex and not very revealing. Here, I will discuss briefly the motion of positrons inside a horizontal toroidal sector. In the system of

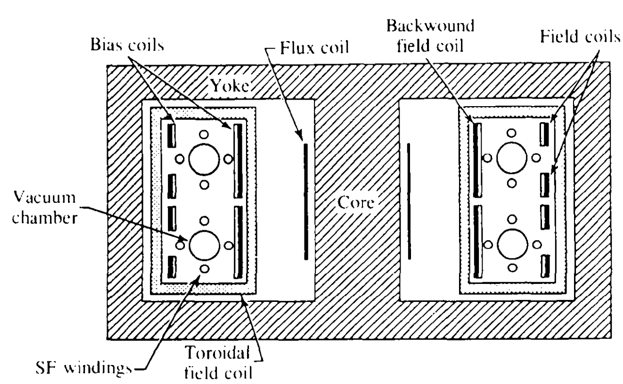

Figure 3
Cross-section of the ZARA-tron.

coordinates shown in Fig. 5, the orbit of the beam centroid that is located inside the z-pinch channel is given by

$$\begin{aligned}\Delta r = \ & \{\Delta \dot{r}(t=0)[\sin \omega_+ t - \sin \omega_- t] \\ & - \Delta \dot{z}(t=0)[\cos \omega_+ t - \cos \omega_- t] \\ & + \Psi_p[\omega_- \cos \omega_+ t - \omega_+ \cos \omega_- t]\}/[(\Omega_{\theta_0}/\gamma_0)^2 + 4\omega_0^2]^{1/2} \\ & + \Psi_p \end{aligned} \quad (1)$$

and

$$\begin{aligned}\Delta z = \ & \{\Delta \dot{r}(t=0)[\cos \omega_+ t - \cos \omega_- t] \\ & + \Delta \dot{z}(t=0)[\sin \omega_+ t - \sin \omega_- t] \\ & - \Psi_p[\omega_- \sin \omega_+ t - \omega_+ \sin \omega_- t]\}/[(\Omega_{\theta_0}/\gamma_0)^2 + 4\omega_0^2]^{1/2}, \end{aligned} \quad (2)$$

where $\Delta r = r - r_0$, r_0 is the major radius, $\Delta \dot{r}(t = 0)$ and $\Delta \dot{z}(t = 0)$ are the transverse velocity components at the target, Ω_{θ_0} and Ω_{z_0} are the cyclotron frequencies of B_θ and B_z, $\omega_0^2 = \omega_r^2 = \omega_z^2 = (\Omega_{z_0}/\gamma_0)^2[2(I_c/I_A)(r_0/r_c)^2]$, I_c is the z-pinch current, r_c is the channel radius, $I_A = 17 \times 10^3\gamma_0\beta_0$, γ_0 is the relativistic factor, $\omega_\pm = (\Omega_{\theta_0}/\gamma_0) \pm [(\Omega_{\theta_0}/2\gamma_0)^2 + \omega_0^2]^{1/2}$, $\Psi_p = (c^2/\omega_0^2 r_0)(\delta\gamma_0/\gamma_0)$ is the particular solution, and $\delta\gamma_0$ is the energy mismatch. Equations (1) and (2) have been derived under the assumption that the self and induced fields of the positron beam, the external field index n and the initial positrons $\Delta r(t = 0)$ and $\Delta z(t = 0)$ are zero.

As a result of their energy spread, the positrons cannot be matched to the fields and thus oscillate around a centre that is located near the symmetry axis of the pipe. According to (1), the centre of the orbit is displaced from the symmetry axis by $\Delta r_0/r_0 = \Psi_p/r_0 \simeq (\delta\gamma_0/\gamma_0)/[2(I_c/I_A)(r_0/r_c)^2]$. For $r_0 = 200$ cm, $r_c = 2$ cm, $\gamma_0 = 7$, $I_c = 10^4$ A and $\delta\gamma_0/\gamma_0 = 0.17$, $\Delta r_0 = 2 \times 10^{-2}$ cm, *i.e.* the displacement is very small. Similarly, the maximum amplitude of the oscillatory orbit is given approximately by $\Delta \dot{r}(t = 0)/2\omega_0$, which for the parameters given above is $2.4 \times \Delta \dot{r}(t = 0)/c$ cm and is small for reasonable values of $\Delta \dot{r}(t = 0)$. For electrons, $I_A = (mc^3/e)\gamma_0\beta_0$ and thus ω_0^2 becomes negative, resulting in complex $\omega_\pm$ whenever $4|\omega_0^2| > (\Omega_{\theta_0}/\gamma_0)^2$. In order to confine the electrons the direction of I_c has to be reversed.

Since the current of the positron beam is approximately two orders of magnitude lower than the current of the electron beam, neither B_θ nor the strong focusing field are necessary to confine the positrons. Both these fields, however, have been retained in the lower torus because they make the system substantially more tolerant to energy mismatch and spread and therefore improve the trapping of positrons.

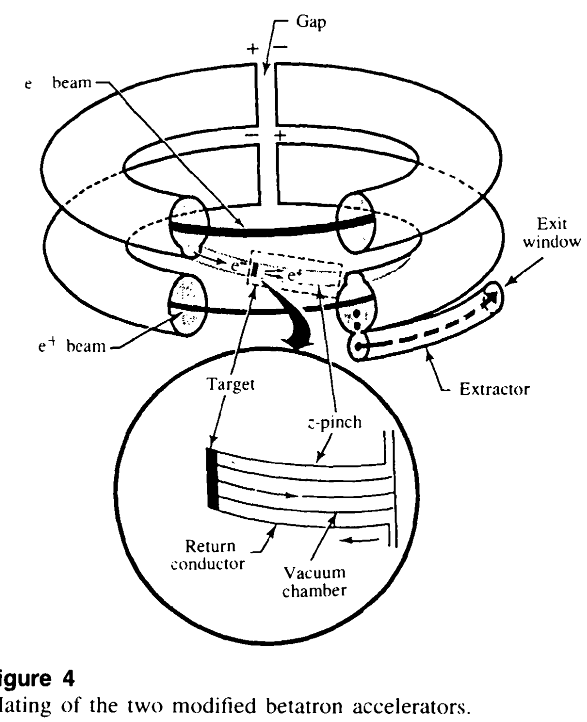

Figure 4
Mating of the two modified betatron accelerators.

5. Point design

The parameters of a point design ZARA-tron are shown in Table 1. The peak energy of 125 MeV for electrons and positrons has been selected rather arbitrarily. Similarly, the repetition frequency of the device is, to a large extent, arbitrary, although consistent with the commercially available thicknesses of the ferromagnetic material. For c.w. operation, the high repetition frequency results in high electron beam average power which is detrimental for the high-Z target. However, for most applications c.w. operation is neither necessary nor desirable.

The peak magnetic flux, φ_p, required to attain the peak energy of 125 MeV is 2.6 Wb. The change in flux $\Delta\varphi$ induced by the self-magnetic field of the beam after it diffuses out of the torus is 8×10^{-4} Wb, *i.e.* very small, and thus it can be neglected. However, modern magnetic materials, such as metglas, require a very small magnetizing force to bring the core to saturation, and thus the self field of the beam will require compensation by a set of conductors that run along the torus having a poloidal distribution that closely resembles the distribution of wall currents in the torus.

A device with the parameters listed in Table 1 is capable of generating $\sim 5 \times 10^{14}$ electrons per pulse. Assuming a

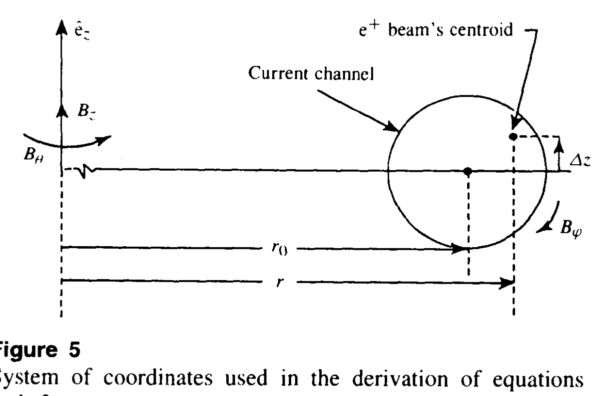

Figure 5
System of coordinates used in the derivation of equations (1) and (2).

Table 1
Point design.

Major radius, r_0	2 m
Peak beam energy	125 MeV
Electron beam current	2 kA
Peak vertical field, B_{z0}	2095 G (a.c. + d.c.)
Toroidal magnetic field, B_{θ_0}	8 kG
Peak magnetic force, H_p	7962 A-turns m^{-1}
Induction at peak current, B_p	19 kG
Material	3% Si strip
Core radius	0.66 m
Peak current in flux coil	10.5 kA
Flux coil inductance	0.25×10^{-3} H
Frequency	1 kHz
$\langle P \rangle$ for (e^- beam)	10 MW

figure of merit 0.05 trapped positrons per electron per GeV, the predicted number of positrons per pulse is 3×10^{12}. At the peak energy of 125 MeV, the positron beam would have an average power of 60 kW. These numbers should be considered speculative because the figure of merit is based on experimental results from linear accelerators and presently very little is known about the emittance of the positron beam at the output end of the z-pinch.

The proposed accelerator has some interesting features, such as intellectual elegance, compactness, a smaller ferromagnetic material requirement than two modified betatron accelerators, and high efficiency, since it utilizes fully the accelerating pulse. Important outstanding issues are the generation of desired fields and, in particular, the cross-talking of bias fields, the cooling of the target, and the transport of the beam from the upper to the lower torus.

References

Barletta, W. A. (1993). *Proceedings of the 1993 IEEE Particle Acceleration Conference*, Washington, DC, pp. 1988–1992.

Barton, M. A. (1985). *IEEE Trans. Nucl. Sci.* **32**, 3350–3353.

Gluckstern, R. L. (1979). *Proceedings of the 1979 Linear Accelerator Conference*, pp. 245–249. Brookhaven National Laboratory, Associated Universities, Upton, New York.

Hasnain, S. S., Helliwell, J. R. & Kamitsubo, H. (1994). *J. Synchrotron Rad.* **1**, 1–4.

Ishizuka, H., Prohasko, R., Fisher, A. & Rostoker, N. (1988). *Proceedings of the 7th International Conference on High-Power Beams*, Vol. 2, pp. 857–863. Kernforschungszentrum Karlsruhe HmbH, Karlsruhe, Germany.

Kapetanakos, C. A., Drakakis, E., Xenidi, Z. & Karabourniotis, D. (1995). *Phys. Rev. Lett.* **75**, 4851–4854.

Kapetanakos, C. A., Len, L. K., Smith, T., Marsh, S. J., Loschialpo, P., Dialetis, D. & Mathew, J. (1993). *Phys. Fluids*, **B5**, 2295–2313.

Kerst, D. W., Adams, G. D., Koch, H. W. & Robirson, L. S. (1950). *Rev. Sci. Instrum.* **21**, 462–480.

Roberson, C., Mondelli, A. & Chernin, D. (1983). *Phys. Rev. Lett.* **50**, 507–510.

Sprangle, P. & Kapetanakos, C. A. (1978). *J. Appl. Phys.* **49**, 1–6.

Stiening, R. (1987). *Proceedings of the 1987 IEEE Particle Acceleration Conference*, Washington, DC, Vol. 1, pp. 1–7.

Wurtel, J. S. (1994). *Phys. Today*, (July), 33–40.