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Normal tracing deflectometry using a secondary light source

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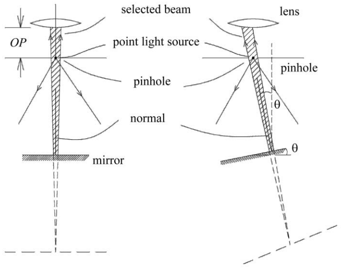

Scanning deflectometric profilers based on an f–θ system are typical optical tools used to measure mirror profiles at many synchrotron facilities. Unlike these profilers, which are based on a pencil beam, here a secondary light source and a pinhole are used to construct a system that automatically selects a beam that will always pass through the pinhole and propagate along the normal direction of the measured area on the surface under test. By measuring the angle variation of the selected beam, slope variations of the surface under test can be measured. Systematic errors introduced by manufacturing defects or aberrations of an optical element, which greatly degrade the performance of traditional profilers, could be minimized by using the developed method. Simulation values of the proposed method and a conventional method are compared.

1. Introduction

X-ray mirrors are important optical elements in synchrotron facilities, X-ray free-electron lasers and astronomy telescopes. The performance of these instruments directly depends on the quality of the X-ray mirrors. Deflectometric instruments, for example the Long Trace Profiler (LTP) (Takacs *et al.*, 1987, 1989; Irick & McKinney, 1992; Irick, 1994; Qian *et al.*, 2013, 2014; Takacs & Qian, 2004; Ritucci & Rossi, 2013), the Nano-Optic Measuring Machine (NOM) (Siewert *et al.*, 2004, 2012), the EADS system (Schulz *et al.*, 2010*a*), the ESAD system (Schulz *et al.*, 2010*b*), the Development Long Trace Profiler (DLTP) (Lacey *et al.*, 2014) and the Stitching Shack Hartmann Optical Head (SSHO) (Idir *et al.*, 2014), are important for the metrology of X-ray mirrors with an accuracy better than hundreds of nanoradians. In the past 20 years, many attempts have been made to improve the performance of these deflectometric instruments (Qian *et al.*, 2014; Yashchuk *et al.*, 2010; Lammert *et al.*, 1997; Yashchuk, 2006, 2009; Siewert *et al.*, 2010). To meet the state-of-art metrology requirements of third-generation X-ray light sources and X-ray free-electron lasers with a metrology accuracy better than 50 nrad, especially for strongly curved mirrors, the metrology ability of these deflectometric instruments needs to be upgraded.

The principle of these deflectometric instruments involves scanning a sample beam along a surface under test (SUT); the angle of the reflected beam varies by 2θ if the slope of the SUT varies by θ. The quality of the SUT can be assessed according to the measured angle variation. As the slope of the SUT varies, the reflected beam from the SUT will have lateral motions on different optical elements (Qian *et al.*, 2013).

Aberrations of the f–θ system of traditional deflectometric profilers will deviate the position of the beam spot on the detector from the position that satisfies the f–θ relation and introduce angle errors to the measurement. Aberrations of an f–θ system cannot be eliminated thoroughly even if we have every optical element of the f–θ system manufactured exactly as designed. Because of beam lateral motions, aberrations of the f–θ system will also introduce lots of systematic errors to the measurement. Theoretically, the high-accuracy angle measurement range of a deflectometric profiler is decreased by aberrations of its f–θ system.

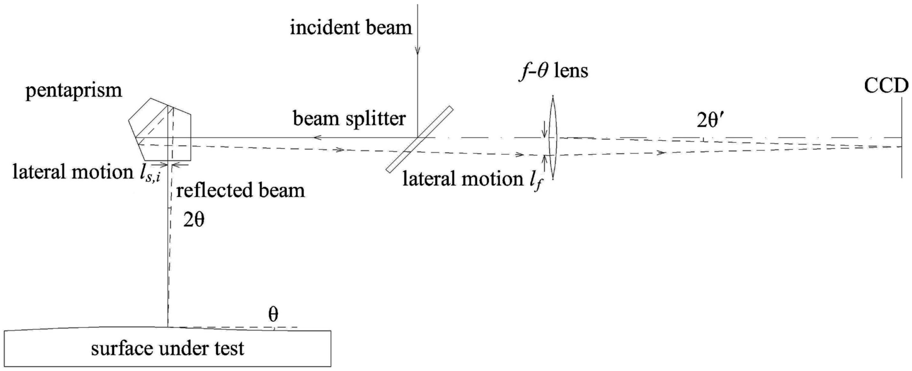

Figure 1
Schematic of a pencil-beam deflectometric profiler.

Manufacturing defects like fabrication imperfections and inhomogeneity of bulk materials in the optical elements of a deflectometric profiler might also introduce systematic errors of hundreds of nanoradians to the measurement because of beam lateral motions. Many strategies have been proposed to minimize this kind of systematic error. Qian *et al.* (2013) developed an advanced nano-accuracy surface profiler to decrease beam lateral motions on the system optical elements so as to lower systematic errors introduced by manufacturing defects in these optical elements. Barber *et al.* (2011) replaced the bulk pentaprism with an optimally aligned mirror-based pentaprism which eliminated systematic errors introduced by inhomogeneity of the optical material and fabrication imperfections of the bulk pentaprism.

The bigger the beam lateral motions are, the larger the systematic errors that might be introduced to the measurement by the optical elements. It is difficult for traditional deflectometric profilers to minimize beam lateral motions on its optical elements because a reasonable working distance is always needed for the SUT (Qian *et al.*, 2013). Traditionally, these systematic errors have been decreased by calibration (Yashchuk *et al.*, 2007). However, because of the distance and angle dependence of these systematic errors, it is challenging to calibrate these systematic errors.

In this paper, to increase the high-accuracy angle measurement range of a deflectometric profiler and to minimize the systematic errors introduced by optical elements, we introduce a normal tracing (NT) method using a secondary light source (SLS) for the construction of new types of deflectometric profilers. Using this NT method, systematic errors caused by beam lateral motions of the optical elements of a deflectometric profiler can be minimized.

2. Deflectometric method

2.1. Analysis of systematic errors introduced by optical elements

Before introducing the NT method, we consider the systematic errors introduced by different optical elements in a deflectometric profiler.

Fig. 1 illustrates the principle of a pencil-beam deflectometric profiler. An incident beam with an aperture of a few millimetres is guided along the optical axis of the f–θ system to the SUT and reflected by the SUT with an angle deviation of 2θ from the optical axis. When the slope angle θ of the SUT varies, the reflected beam will pass through a different part of the optical element. Manufacturing defects and aberrations of the optical system will thus introduce systematic errors into the measurement. Working with a beam with an aperture of a few millimetres revealed that only low-frequency systematic errors were introduced into the measurement.

When only systematic errors introduced by optical elements are considered, the measured angle θ' of a pencil-beam deflectometric profiler at a position on an SUT could be represented as

$$\theta' = \theta + \sum_i \varepsilon_s^i + \varepsilon_f, \qquad (1)$$

where ε_s^i is the angle error introduced by manufacturing defects of the ith optical element and ε_f is the angle error introduced by aberrations of the f–θ system. ε_s^i and ε_f are functions of the lateral motions of the reflected beam from the optical axis and could be represented by a series expansion of lateral motions as

$$\varepsilon_s^i = \sum_{m=0}^{\infty} A_m^i \, l_{s,i}^m \qquad (2)$$

and

$$\varepsilon_f = \sum_{m=1}^{\infty} B_m \, l_f^{2m-1}. \qquad (3)$$

For simplicity, only the scanning optical head modes like LTP II (Qian & Qian, 2010) were considered. In this case, the distances from the SUT to optical elements of the profiler were not changed. Here, constants A_m^i are expansion coefficients of the ith optical element and B_m are expansion coefficients of the f–θ lens system, $l_{s,i}$ are lateral motions of the reflected beam on the first incident surface of the ith optical element and l_f are lateral motions of the reflected beam on the first incident surface of the f–θ lens system. Only odd terms exist in equation (3) because of the sign conventions of the optical system and aberrations of the lens that are rotationally symmetric to the optical axis.

From equation (3), we can see that systematic errors introduced by aberrations of the optical system should be rotationally symmetric about the origin of the reference. This might be the only known source that could introduce such a kind of systematic error.

To characterize the profile of a SUT, only the angle difference $\Delta\theta$ at different scanning points on the SUT is needed,

$$\Delta\theta' = \Delta\theta + \sum_i \Delta\varepsilon_s^i + \Delta\varepsilon_f. \tag{4}$$

If variations of lateral motions on different optical elements are very small, systematic errors introduced by different optical elements could be simplified as

$$\Delta\varepsilon_s^i = \sum_{m=1}^{\infty} m\, A_m^i\, l_{s,i}^{m-1} \Delta l_{s,i}, \tag{5}$$

$$\Delta\varepsilon_f = \sum_{m=1}^{\infty} (2m-1) B_m\, l_f^{2m-2} \Delta l_f. \tag{6}$$

Here, $\Delta l_{s,i}$ and Δl_f are beam lateral motions of the reflected beam at different optical elements and beam lateral motion on the first optical element of the f–θ system, respectively.

Fig. 2 revels that when the angle between the beam and optical axis ψ is very small, the lateral motion l of a beam from the optical axis on a surface of an optical element could be calculated as

$$l = OP \times \psi, \tag{7}$$

where OP is the optical path distance from the intersection point of the beam and optical axis to the first surface of an optical element. The beam lateral motions Δl could be represented as

$$\Delta l = \Delta OP \times \psi \;+\; OP \times \Delta\psi. \tag{8}$$

For a deflectometric profiler like that shown in Fig. 1, ΔOP of the first term in equation (8) might be hundreds of millimetres when a SUT with a length of hundreds millimetres is being tested. To minimize the first term of Δl of a deflectometric profiler like that in Fig. 1, the SUT should be placed horizontally to minimize ψ or the scanning optical head method must be used like in LTP II (Qian & Qian, 2010) to minimize ΔOP. In the second term of equation (8), $\Delta\psi$ is a function of $\Delta\theta$ and difficult to minimize. The only way to minimize the second term of Δl is to minimize OP. Considering the set-up in Fig. 1, to minimize the second term of Δl, the optical elements of the deflectometric profiler should be placed as close as possible to the intersection point of the incident beam on the SUT, otherwise small beam lateral motion or high-accuracy angle metrology could only be realised within a small angle measurement range.

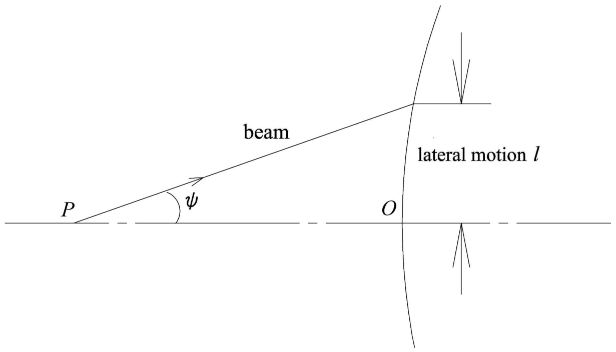

Figure 2
Lateral motion of a beam from the optical axis on the surface of an optical element.

To decrease systematic errors introduced by manufacturing defects in an optical element, we can decrease beam lateral motions on the optical element using a novel design of the deflectometric profiler or have optical elements accurately manufactured with high-quality materials to minimize systematic errors introduced by manufacturing defects. In contrast, systematic errors caused by aberrations are different. Because aberrations of an optical system cannot be minimized thoroughly by any optical design process, even if all optical elements are manufactured perfectly as designed, aberrations of the optical system still introduce systematic errors into the measurement. Theoretically, the high-accuracy angle measurement range of a deflectometric profiler is restricted by aberrations of its optical system. To decrease aberration-introduced systematic errors, the optical system should be well designed to decrease aberrations or the beam lateral motions on the f–θ system should be decreased.

The above analysis reveals that systematic errors introduced by optical elements could be minimized by minimizing beam lateral motions on the optical element. For traditional deflectometric profilers with an angle measuring range of about a few tens of milliradians, the only way to minimize the beam lateral motions on an optical element is to place the optical element as close as possible to the intersection point of the incident beam on the SUT. However, to protect a valuable SUT, a reasonable working distance from the nearest system optical element to the SUT is always needed. Thus, it is difficult for traditional deflectometric profilers to minimize beam lateral motions on its optical elements. When the optical path distance from the intersection point to an optical element is hundreds of millimetres or longer, high-accuracy angle metrology could only be realised within a small angle range.

2.2. Normal tracing (NT) method using a secondary light source

To construct a new type of deflectometric profiler with minimal beam lateral motions on different system optical elements, we developed the NT method.

Fig. 3 illustrates the NT method based on a point light source. From Fig. 3 we can see that, when a point light source is located at the centre of a pinhole, rays emitted from the point light source and reflected by the mirror are selected by the pinhole. The selected rays that could pass through the pinhole form a conical beam that propagates along the normal direction of the mirror with a small cone angle. The intersection spot of the selected beam formed a measured area on the mirror. Using this set-up, we can always select a beam that propagates along the normal direction of the measured area on the mirror with a small cone angle. If the angle of the

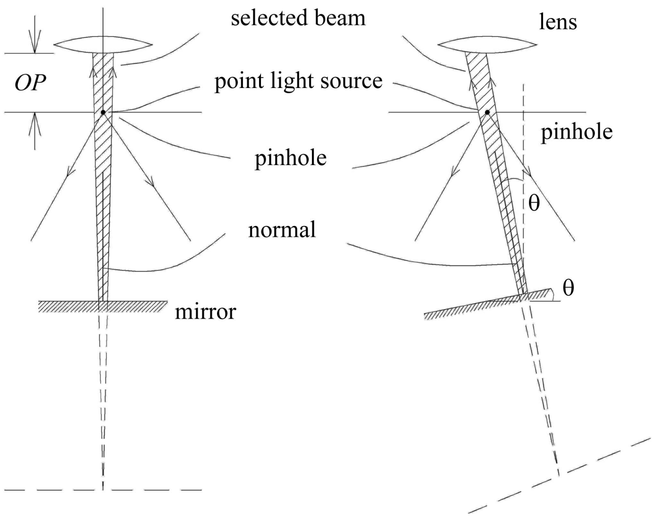

Figure 3
Schematic of the NT method based on a point light source. *OP* is the optical path distance used to calculate the beam lateral motion and θ is the angle variation of the mirror.

mirror varies by θ, the selected beam will have the same angle variation of θ. The cone angle of the selected beam is determined by the distance from the pinhole to the mirror, the aperture of the pinhole and the curvature of the measured area on the mirror. By measuring the angle variation of the selected beam, the angle variation of the mirror could be determined.

Fig. 3 also reveals that the *OP* value used to calculate beam lateral motions on an optical element (a lens in Fig. 3) was from the centre of the pinhole to the optical element. ΔOP in the first term of equation (8) is zero when *OP* does not change during the measurement, so the first part of equation (8) could be minimized. The second term of equation (8) could also be minimized by placing the optical element as close as possible to the pinhole to minimize *OP*. As a result, beam lateral motions on the optical element could be minimized. However, it is difficult to place a point light source in the centre of a pinhole, especially when the aperture of the pinhole might be a few millimetres or hundreds of micrometres. A point light source located in the centre of the pinhole could be realised by using a SLS.

Fig. 4 depicts our NT method using a point SLS. The SLS is the reflection image of a point light source induced by a beam splitter (BS). By appropriately arranging the relative positions of the pinhole, BS and point light source, rays emitted from the point light source and reflected by the BS could be treated as rays emitted from the SLS, which is located at the centre of the pinhole. Because the BS could be placed as close as possible to the pinhole, systematic errors introduced by the BS might be very small.

For deflectometric profilers designed based on the NT method, their optical elements could be placed as close as possible to the pinhole. By doing this, small beam lateral motions on the optical elements of a deflectometric profiler designed based on the NT method could be realised within a large angle measurement range, and we could also have a long working distance for the SUT. That is, deflectometric profilers designed based on the NT method could display a large angle measurement range of high-accuracy metrology and systematic errors introduced by different optical elements could be minimized.

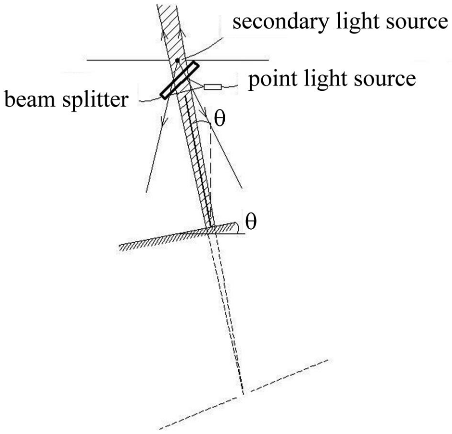

Figure 4
Schematic of the NT method using a point secondary light source.

2.3. Properties of deflectometric profilers based on the NT method

Deflectometric profilers designed based on the NT method will have many different properties from those of traditional deflectometric profilers.

2.3.1. NT method for the metrology of strongly curved mirrors. To obtain high-spatial-frequency information of a SUT, we need the measured area on the SUT to be as small as possible. In the NT method, the aperture of the measured area depends on the radius of curvature of the measured area. For example, as illustrated in Fig. 5(*a*), if the SUT has a spherical surface with a radius of curvature equal to the distance from the pinhole to the mirror h, the beam spot might be the largest on the SUT, which is not acceptable. To measure high-spatial-frequency information of the SUT, the size of the measured area on the SUT must be restricted.

In Fig. 5, Φ_s is the cone angle of the point light source, and d_{ma}, h and d_{ph} are the aperture of the measured area, distance from the pinhole to the mirror, and aperture of the pinhole, respectively. Suppose the measured area on the SUT could be treated as a spherical surface with a radius of curvature R, the aperture d_{ma} of the measured area on the SUT should be no bigger than the aperture of the pinhole d_{ph}. In the tangent plane, the line of intersection of the measured area could be treated as part of a circle with R and the slope of the line of the measured area could be represented as

$$y' = \frac{x}{(R^2 - x^2)^{1/2}}. \tag{9}$$

The origin of the coordinate system is located at the centre of the circle. The x coordinate is horizontal, the y coordinate is vertical and the system is right-handed. If $d_{ma} = d_{ph}$ and $x =$

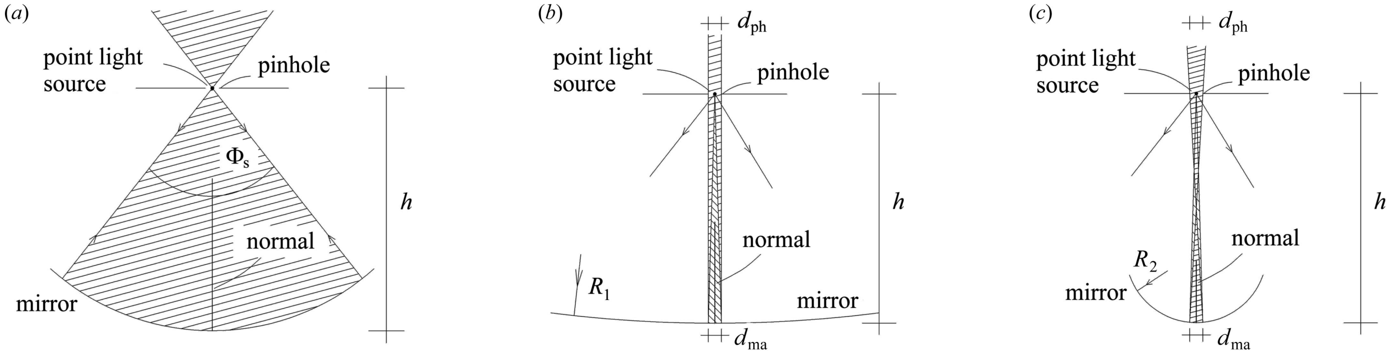

Figure 5
Relationship between the aperture and curvature of the measured area. (*a*) Measured area with the largest aperture. (*b*) The first case, where the aperture of the measured area equals the aperture of the pinhole and the radius of curvature of the measured area is R_1. (*c*) The second case, where the aperture of the measured area equals the aperture of the pinhole and the radius of curvature of the measured area is R_2.

$d_{ph}/2$ (Figs. 5*b* and 5*c*), the intersection point on the mirror of the ray with the biggest cone angle of the selected beam will have a slope of y_1' for the case shown in Fig. 5(*b*) or y_2' for that in Fig. 5(*c*),

$$y_1' = d_{ph}/4h, \tag{10}$$

$$y_2' = 3d_{ph}/4h. \tag{11}$$

The radius of curvature R_1 and R_2 described in Figs. 5(*b*) and 5(*c*), respectively, could be calculated as

$$R_1 = (1/2)\left(16h^2 + d_{ph}^2\right)^{1/2}, \tag{12}$$

$$R_2 = (1/6)\left(16h^2 + 9d_{ph}^2\right)^{1/2}. \tag{13}$$

Therefore, if R of the measured area is bigger than R_1 or smaller than R_2, we will have a measured area on the SUT with an aperture smaller than that of the pinhole. For example, if the aperture of the pinhole d_{ph} is 1 mm and h is 500 mm, R_1 = 1 m and R_2 = 0.333 m, as calculated using equations (12) and (13), respectively. In this case, if we want the aperture of the measured area on the SUT to be smaller than 1 mm, all the possible measurable areas on the SUT with an aperture of 1 mm should have an R bigger than 1 m or smaller than 0.333 m. From this point of view, the NT method might be a good approach for the metrology of very strongly curved mirrors.

2.3.2. Relationship between the position of the measured area and slope of the SUT. For traditional deflectometric profilers the measured area on a SUT is the intersection spot of the incident beam and is directly determined by the incident beam. In the NT method the measured area on a SUT is an area with its normal pointed to the centre of the pinhole. Therefore, the exact position of the area on a SUT measured by the NT method needs to be calculated according to h and the measured angle.

In a measurement trace, only the relative positions of measured areas on a SUT are important for the metrology. To calculate the relative position of a measured area, we need to choose one measured area as a reference point. For example, in Fig. 6, if we choose position p_1 as the reference point, the distance d_{pp} from position p_1 to p_2 could be roughly calculated from the centre deviation of the pinhole d_{cc}, h and the angle difference β of the two positions as

$$d_{pp} = d_{cc} + h\beta. \tag{14}$$

The distance h could be obtained by a high-accuracy laser range finder. However, when β between the measured and reference points is very large, the distance calculated by equation (14) might not be accurate. Fig. 6 reveals that h is not accurate for calculating the distance and it might be corrected as

$$h' = h\left(1 - \beta^2 + \beta^4 - \beta^6 + \ldots\right) = h/\left(1 + \beta^2\right). \tag{15}$$

The angle error $\Delta y'$ introduced by position deviation Δx could be derived from equation (9) as

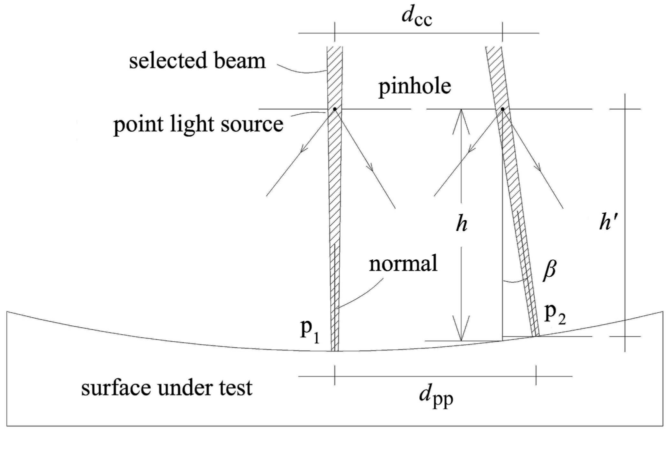

Figure 6
Position of the measured area on a SUT using the NT method. Here, p_1 and p_2 are measured positions on the SUT, and d_{cc}, d_{pp}, h, h' and β are the pinhole distance of the measured area at position p_1 and p_2, the horizontal distance from p_1 to p_2, the distance from the pinhole to the SUT, the distance for calculating the precise position of the measured area, and the slope difference of p_1 and p_2, respectively.

$$\Delta y' = \left[\frac{x^2}{\left(R^2 - x^2\right)^{1.5}} + \frac{1}{\left(R^2 - x^2\right)^{1/2}}\right]\Delta x$$
$$= \left[\frac{\theta^2}{R\left(1 - \theta^2\right)^{1.5}} + \frac{1}{R\left(1 - \theta^2\right)^{1/2}}\right]\Delta x. \qquad (16)$$

Here, $\theta = x/R$ is the measured angle. Within the measurement range of ±10 mrad, $\Delta y' \simeq \Delta x/R$. If Δx is only introduced by the distance deviation Δh and $\Delta y'$ should be no bigger than 50 nrad, we will have

$$\Delta x = \Delta h \beta, \qquad (17)$$

$$\Delta y' = (\Delta h \beta)/R = 5 \times 10^{-8} \qquad (18)$$

and

$$\Delta h = (R/\beta) \times 5 \times 10^{-8}. \qquad (19)$$

In equation (19), if β is the biggest measureable angle, 10 mrad, and $R > 1000$ m, Δh could be a few millimetres. For a strong curved mirror, h should be precisely measured.

2.3.3. Systematic error introduced by the position deviation of the point light source from the centre of the pinhole.

The selected beam in the NT method could be treated as a beam that always passes through the pinhole. The direction of this beam could be represented by a ray that is emitted from the point light source, reflected back by the SUT and passes through the centre of the pinhole, as illustrated in Fig. 7. If the position of the point light source deviates from the centre of the pinhole, the selected beam might deviate from the normal of the measured area on the mirror.

Fig. 7 shows that different kinds of deviations of the point light source from the centre of the pinhole will introduce different angle errors to the selected beam. However, it should be noted that only the difference of measured angles is relevant to metrology. When the mirror is horizontally placed, we define the selected beam as the reference beam with its measured angle of zero. After the mirror is tilted by angle θ, the angle difference of the selected beam from the reference beam is θ' (Fig. 7) and the angle errors could be defined as

$$\Delta\theta = \theta - \theta'. \qquad (20)$$

Fig. 8 indicates that vertical position deviation will introduce non-negligible angle errors to the measurement. If we can accurately place the point light source at the centre of the pinhole, the angle errors introduced by position deviation could be minimized. For example, if h = 500 mm and the vertical position deviation from the SLS to the centre of the

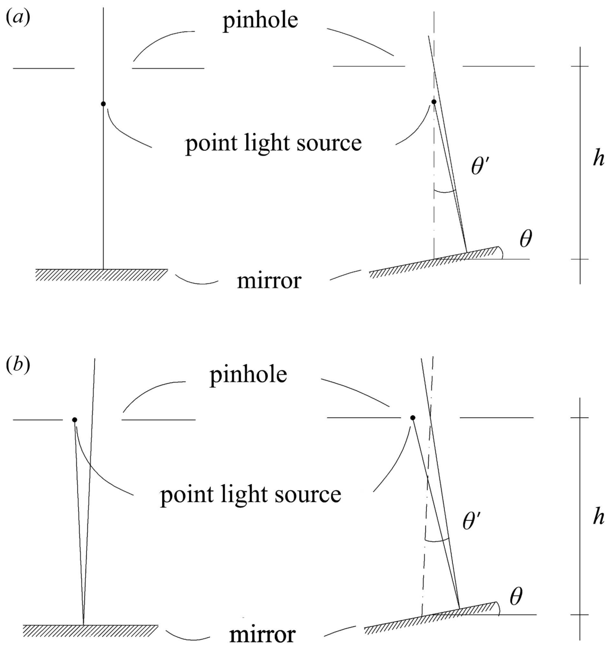

Figure 7
Position deviation of the point light source from the centre of the pinhole. (*a*) Vertical position deviation of the point light source from the centre of the pinhole. (*b*) Horizontal position deviation of the point light source from the centre of the pinhole. θ is the angle variation of the mirror and θ' is the measured angle.

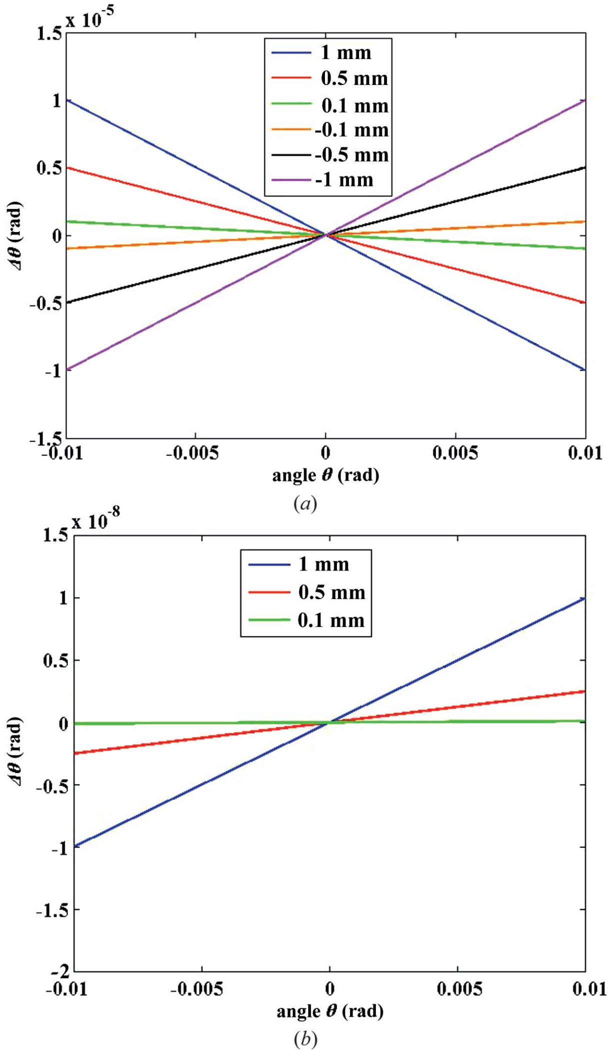

Figure 8
Angle errors of different position deviations of the point light source from the centre of the pinhole within an angle measurement range of ±10 mrad with h = 500 mm. (*a*) Vertical position deviations from −1 to +1 mm of the point light source from the centre of the pinhole. (*b*) Horizontal position deviations from 0.1 mm to 1 mm of the point light source from the centre of the pinhole.

pinhole is about 10 µm, an angle error smaller than 50 nrad might be obtained within an angle range of 10 mrad. If it is difficult to place the point light source at the centre of the pinhole, because of its linear dependence between $\Delta\theta$ and θ (Fig. 8) it can be easily calibrated.

2.3.4. Loss of luminous energy. According to the NT method, the selected beam is just a small part of the rays from the point light source. If rays emitted from the point light source could be treated as a spherical wave within cone angle Φ_s with power w_s (Fig. 9), the luminous energy of the selected rays could be calculated by comparing the solid angle of the selected rays and rays emitted from the point light source.

The solid angle of rays emitted from a point light source within Φ_s could be represented as

$$\Omega_s = 2\pi\left[1 - \cos(\Phi_s/2)\right]. \tag{21}$$

To measure high-spatial-frequency information of a SUT, the measured area on a SUT might have a d_{ma} of a few millimetres or even less. To protect the SUT, h might be hundreds or thousands of millimetres. Therefore, the cone angle Φ_{ma} of the selected rays could be simply represented as

$$\Phi_{ma} = d_{ma}/h. \tag{22}$$

The solid angle of the selected rays could be approximately calculated as

$$\Omega_p = 2\pi \int\limits_0^{d_{ma}/2h} \sin\varphi \, d\varphi = 2\pi\left[1 - \cos(d_{ma}/2h)\right]. \tag{23}$$

Thus, the effective luminous energy w_p of the selected rays is

$$w_p = w_s \, \frac{1 - \cos(d_{ma}/2h)}{1 - \cos(\Phi_s/2)}. \tag{24}$$

Φ_s must be bigger than the designed angle measurement range of a deflectometric profiler. Equation (24) shows that to increase the luminous energy of the selected beam we can use a high-power point light source and restrict its Φ_s reasonably according to the angle measurement range. Although increasing d_{ph} of the pinhole and minimizing h could also increase the effective luminous energy, modifying these parameters is not suggested. Enlarging d_{ph} might increase the aperture of the measured area of the selected beam on the SUT d_{ma} and lose high-spatial-frequency information of the SUT. Minimizing h is also not advised because a reasonable working distance for the SUT is needed, and it might increase the aperture of the beam spot of selected rays on the SUT.

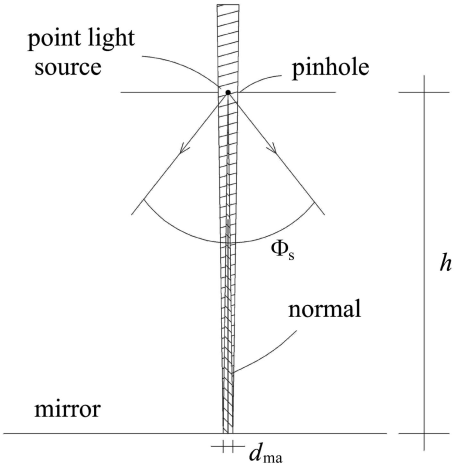

Figure 9
The angle relation of luminous energy of the selected rays from a point light source.

2.3.5. Effect of actual light source size on selected beam. A point light source is an ideal mathematical model for the simulation of an actual small-sized light source. An actual small light source could be treated as many point light sources scattered on the surface of the light source. The selected beam by the pinhole of an actual small light source cannot be treated as a conical beam. In this case, it is a beam that passes through the centre of the pinhole with small angle divergence.

As we pointed out in §2.3.3, the horizontal position deviation of the point light source will introduce minimal angle errors to the measurement, which means the selected beam of an actual small light source will also be a beam that is sensitive to the slope variation of the SUT. By measuring the angle differences of the selected beam, slope differences of the SUT could be measured.

However, for a small light source, the measured area on the SUT must be bigger than that of the point light source. If the aperture of the measured area on the SUT is d_{ma} for a point light source in the NT method and the size of the actual light source is d_s, the aperture of the measured area on the SUT might be $d_{ma} + d_s$. For a source with a size d_s of a few micrometres (*e.g.* an optical fibre), the effect of the size of the actual light source on the aperture of the measured area is negligible.

If the size of the light source is very big, for example a surface light source that is bigger than the aperture of the pinhole, and if we put this light source above the pinhole, we can also select a beam from this set-up.

Fig. 10 illustrates a surface light source and pinhole that make up a beam select system. Rays emitted from the surface light source are selected by the pinhole, and only a small portion of those rays can reach the mirror. This time, the pinhole could be treated as the SLS. The rays reflected by the mirror are selected again by the pinhole and only a small part of the rays reflected by the SUT could pass through the same pinhole and form a probe beam. By using this set-up we can also select a beam that propagates through the normal direction of the measured area on the mirror with small angle divergence. The divergent angle of the beam is determined by the distance from the pinhole to the mirror, the aperture of the pinhole and the curvature of the measured area on the mirror.

The NT method using a surface light source has many different qualities from those of the NT method using a point light source. For example, there is no need to place the light source at the centre of the pinhole, because the SLS of the NT method using a surface light source is the pinhole itself and the position deviation from the SLS to the pinhole is zero (as described in §2.3.3). In this case, the surface light source could be placed slightly above the pinhole to obtain a selected beam

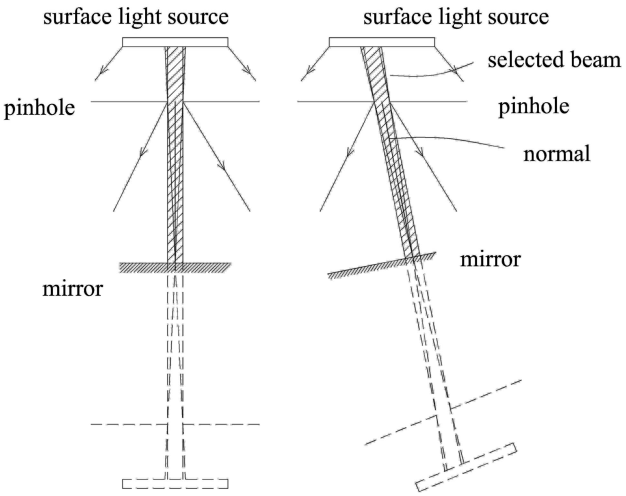

Figure 10
The NT method using a surface secondary light source.

that propagates along the normal of the measured area on the mirror. The measured area on a SUT is different, and is always bigger than that using a point light source.

3. Software simulation

In this section we describe software simulations to illustrate that systematic errors introduced by beam lateral motions on an optical element could be minimized using the NT method. In the software simulations, the experiment environment could be treated as perfect and manufacturing defects of all the optical elements are not considered. For traditional deflectometric profilers, because of aberrations of the f–θ system, systematic errors of up to tens of microradians might be introduced, even in simulations. In our simulation, a simple commercial lens (LA1461, Thorlabls; see Fig. 11) was used to perform the f–θ conversion.

We constructed a verification system (Fig. 12) in the non-sequential mode of Zemax. By appropriately arranging the positions of the pinhole, BS and point light source, rays emitted from the point light source and reflected by the BS could be treated as rays emitted from the centre of the pinhole (Fig. 12). The rays that were reflected by the mirror, passed through the BS and selected by the pinhole formed a probe beam. The LA1461 lens was placed next to the pinhole to minimize the optical distance needed to calculate beam lateral motions on the lens. If the mirror is rotated through an angle θ, the selected beam will be rotated through the same angle.

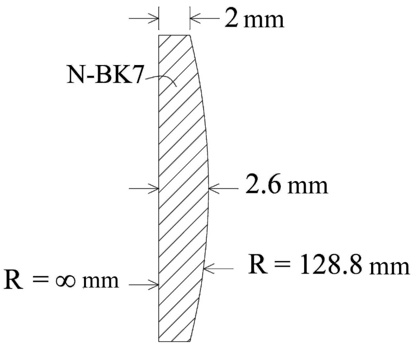

Figure 11
Configuration parameters of lens LA1461 with f = 250.0 mm.

In Fig. 12(a) the optical path from the point light source to the lens and the optical path from the lens to the charge-coupled device (CCD) should satisfy the object–image relation of the lens so that the selected beam can be focused on the CCD. Although the distance from the lens to the mirror was about 250 mm, the distance used to calculate beam lateral motions on the lens was very small. Fig. 12(b) reveals that angle errors smaller than 60 nrad were obtained within the angle range from −5 mrad to +5 mrad.

In Zemax non-sequential simulations, rays are randomly generated by the point light source within the core angle

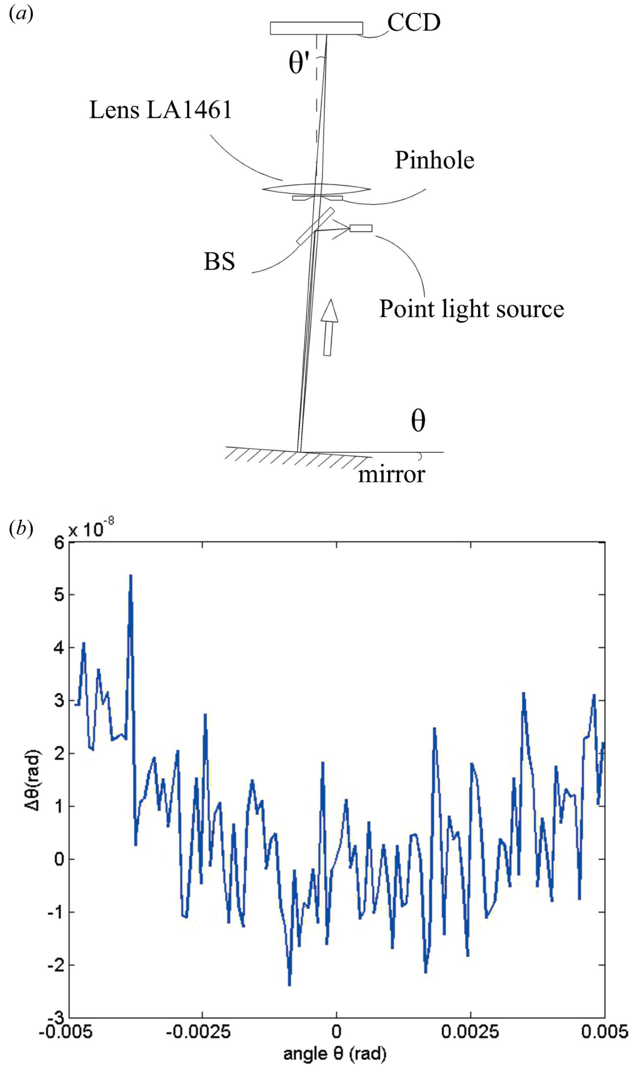

Figure 12
Zemax simulation system of the NT method with a point secondary light source. (a) Schematic of the verification system designed in the non-sequential mode of Zemax. (b) Angle errors $\Delta\theta = \theta' - \theta$ obtained from Zemax simulations within an angle range of ±5 mrad. θ is the angle variation of the mirror, θ' is the measured angle of the NT method and the aperture of the pinhole is 1 mm.

(Zemax, 2011) so that the focal spot in Zemax simulations might deviate slightly from the actual beam spot from time to time. The centre of the beam spot on the CCD was calculated by the centre of mass method. For a CCD with a pixel size of 2 µm within the Zemax simulation, the centre of mass method might not be accurate enough to find the precise spot centre because position deviations of tens of nanometres might be introduced (Canabal *et al.*, 2001). Therefore, it is reasonable to believe that the angle errors smaller than 60 nrad within the angle range from −5 mrad to +5 mrad described in Fig. 12(*b*) were not only introduced by the LA1461 lens.

4. Deflectometric profiler with minimal beam lateral motion designed on the basis of the NT method

Different types of deflectometric profilers could be designed based on the NT method. To design a deflectometric profiler with minimal systematic errors generated by its optical elements, the number of optical elements used in the optical system and the optical path used to calculate beam lateral motions on these elements need to be minimized. In this section, we introduce one possible design of a deflectometric profiler with minimal beam lateral motion based on the NT method with a point light source.

Fig. 13 depicts the design of a deflectometric profiler with minimal beam lateral motion based on the NT method with a point light source. Similar to the simulation described in §3, the positions of the pinhole, BS and point light source should be appropriately arranged so that rays emitted from the point light source and reflected by the BS can be treated as rays emitted from the centre of the pinhole. The rays reflected by the SUT will be restricted by the pinhole; only a small part of these rays pass through the pinhole and form a probe beam. The Fourier transform (FT) lens placed next to the pinhole focuses the probe beam on the CCD. In this design, minimal beam lateral motions on the FT lens could be realised. The BS could be placed as close as possible to the pinhole, so beam lateral motions on the BS could also be very small. The transmitted rays of the point light source through the BS are collimated by a collimating lens and used as a reference beam. The reference beam is propagated along the moving direction of the carriage and measured by an *f*–*θ* system that is mounted on the same stage as the SUT. Because vibrations of the carriage could be very small for a high-accuracy air-bearing stage, beam lateral motions of the reference beam on the *f*–*θ* system could also be very small.

5. Discussion and conclusions

In this paper, we introduced a novel SLS-based NT method for high-accuracy surface profile metrology. Many qualities of the NT method were considered. Based on the NT method, many deflectometric profilers could be designed, and a possible design based on the NT method was developed.

To inspect high-spatial-frequency information of a SUT, a pinhole with an aperture of millimetres or hundreds of micrometres might be used. To minimize the diffraction-introduced systematic errors, a superluminescent broadband light source like a superluminescent light-emitting diode might be a good choice.

Zemax simulations indicated that systematic errors introduced by aberrations of the *f*–*θ* system can be minimized using the NT method. For traditional deflectometric profilers, aberrations of the optical system might introduce systematic errors of thousands of nanoradians to a measurement. For example, angle errors introduced by a FT lens [Fig. E of Zeschke (2012)] and by the NOM optical system [Fig. 3 of Yashchuk *et al.* (2007)] were obtained before or after a simple calibration. As we pointed out in §2.1, aberrations might be the only known systematic error source that could introduce systematic errors rotationally symmetric to the origin of the reference. Although it was not pointed out clearly by Yashchuk *et al.* (2007) and Zeschke (2012), except for the small waves in each curve of the angle error in Fig. E of Zeschke (2012) and Fig. 3 of Yashchuk *et al.* (2007), we believe that systematic errors of up to thousands of nanoradians that are almost rotationally symmetric to the origin of the reference might be introduced by aberrations of the optical system. Traditionally, to minimize these kinds of systematic errors, a sophisticated calibration method is used. Compared with the systematic errors smaller than 60 nrad introduced by a simple commercial lens (LA1461) within a angle measuring range of ±5 mrad in our simulation, uncalibrated systematic errors introduced by aberrations of the *f*–*θ* system have been well minimized.

Zemax simulations revealed that small beam lateral motions on the optical elements and a large working distance for the SUT could be realised simultaneously. Using the NT method, an angle metrology accuracy of tens of nanoradians for an angle measurement range of ∼milliradians is possible. For deflectometric profilers designed based on the NT method, systematic errors

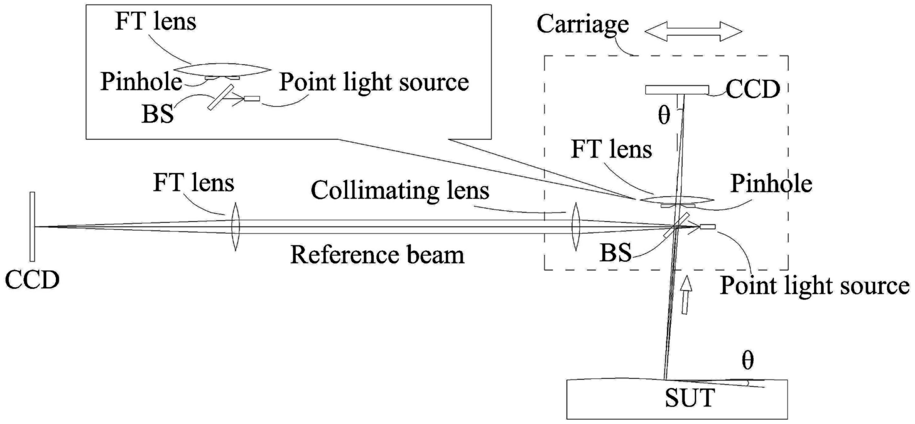

Figure 13
Deflectometric profiler designed based on the NT method with a point light source.

introduced by optical elements could be minimized simply by placing optical elements as close as possible to the pinhole.

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