s1.m1.o5 Cluster covering of dodecagonal and related structures. S.I. Ben-Abraham[1], P. Gummelt[2], R. Lück[3], F. Gähler[4], [1]*Physics Dept., Ben-Gurion Univ., POB 653, IL-84105 Beer-Sheba, Israel* , [2]*Inst. f. Mathematik und Informatik, Arndt-Univ. Greifswald, D-17487 Greifswald, Germany,* [3]*MPI f. Metallforschung, Seestr. 92, D-70147 Stuttgart, Germany,* [4]*Institut f. Theor. u. Angew. Physik, Univ. Stuttgart, D-70550 Stuttgart, Germany e-mail: benabr@bgumail.bgu.ac.il*
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Partially overlapping stable atomic clusters provide a plausible mechanism for the formation and stabilization of quasicrystals. Maximal covering of the space by such clusters creates long range order. This "covering cluster approach" has been well founded mathematically and successfully applied to particular octagonal and decagonal quasicrystals. It is a challenge to extend this approach to other structures, such as dodecagonal.

The centered octagonal quasicrystalline structure can serve as a paradigm of very general structural principles. It is a layer structure of type Š AB´AB´´Š. The A layers have 2n-fold symmetry (in this case, eightfold), the B layers have only n-fold symmetry (here n = 4) and are rotated by 2¼/n (here 45°) with respect to each other. Thus there is a 2nn screw axis (here 84). This is also achieved by coloring the layers and permuting the colors in successive layers. The three-dimensional building block is an AB´AB´´A stack of plane clusters which forms a single kind of overlapping endless prisms. Structures of this kind are very frequent. They may be periodic as well as quasi-periodic. The "covering cluster approach" might thus be as important for the understanding of periodic crystals as it is for quasicrystals. The known dodecagonal quasicrystals (notably found in the NiCr, VNiSi and TaTe alloy systems) also show the mentioned layer structures.

The case of the dodecagonal structures is more subtle. Complete covering of dodecagonal quasiperiodic structures requires two clusters. Yet by slightly relaxing the constraints one may achieve a fair amount of progress with a single one. Even though such an approach is unsatisfactory from a rigorous mathematical point of view it might be quite useful for applications to real quasicrystals.

The known dodecagonal tilings are all related among themselves by local derivability. A single "almost covering cluster" is explicitly presented for the ship, shield, square-triangle and Socolar (alias butterfly) tilings. In all these cases the cluster admits periodic solutions but they are statistically insignificant. However, the periodic solutions might well be preferred on physical, such as energetic, grounds.

The Socolar tiling has an interesting twist. The tiling can be almost covered except for certain hexagons. Nevertheless, all edges and vertices are correctly reproduced. Thus this "almost-covering" might be physically as good as the more intricate complete covering. One the other hand, one observes that the tiling has (in/de)flation symmetry. Hence, the "holes" occur regularly at all levels of the flation. Thus the incomplete covering produces a fractal structure. The fractal dimension can be easily calculated and is d = 1.972 Š.

Notes