Finite-index normal subgroups of crystallographic space groups

Miles A. Clemens¹, Branton J. Campbell¹, Stephen P. Humphries², Harold T. Stokes¹ ¹Department of Physics & Astronomy, Brigham Young University, Provo, Utah, USA ²Department of Mathematics, Brigham Young University, Provo, Utah, USA

Symmetry-lowering phase transitions give rise to crystal domain patterns in ferroelectrics and many other types of materials. If the order parameter responsible for the transition possesses child space-group H, which is a subgroup of the parent space group G, the crystal domains of the child phase are associated with equivalent directions of this order-parameter, which are in one-to-one correspondence with the set S of cosets of H in G. While H is not normal in general, the normal core N of H in G is a finite-index normal subgroup of G, for which the quotient group G/N has a well-defined permutation action on S. In this sense, G/N is the symmetry group of S, and can be used to classify its symmetry-inequivalent domain pairs [1-3]. The importance of finite-index normal subgroups to the study of crystal-domain configurations motivates us to tabulate them for each crystallographic space group.

[1] D. M. Hatch, R. A. Hatt, H. T. Stokes, Ferroelectrics 191, 29-35 (1997).

[2] D. B. Litvin, Acta Cryst. A55, 884-890 (1999).

[3] J. Fuksa, Ferroelectrics 240, 1275-1284 (2000).