Layer Groups associated with 3-way 3-fold isonemal fabrics

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A 3-*fold fabric* denoted by F, consists of three congruent non-parallel layers of strands in a plane E together with a preferential ranking or ordering of the three layers at every point P of F that does not lie on the boundary of a strand, such that F hangs together. The *ranking* must satisfy the fact that if P belongs to a strand T_i of layer i and T_j of layer j ($i \neq j$, $i,j \in \{1,2,3\}$), then if layer i is ranked before j at P, then layer i must be ranked before layer j at every point of $T_i \cap T_j$. The fabric F *hanging together* means it is impossible to partition the set of all strands, belonging to all the layers, into two nonempty subsets so that each strand in the first subset passes over (is ranked before, or takes precedence over) every strand in the second subset. The fabric F is 3-*way*, if the strands lie in three different directions in E [1].

This paper will discuss symmetry groups of 3-way 3-fold fabrics. The symmetry group $S(F)$ of the fabric F is a layer group and consists of isometries of the Euclidean space which map each strand of F onto a strand of F that either preserves the rankings at each point of F (preserves the sides of F) or reverses all the rankings (interchange the sides of F). The approach to describe the symmetry group of F will be to construct a corresponding design of F, $D(F)$ which characterizes the fabric in terms of the rankings of the layers. To represent F, we consider on the plane E of F, sets of equidistant parallel lines to represent the edges (boundaries) of the strands; with lines lying in three different directions. These lines divide E into a set of polygonal regions or tiles, each of which is assigned a color indicating the ranking of the layers at every point of the region or tile. The result is a coloring of a tiling which is called the *design* of F, $D(F)$. An example of a sketch of a 3-way 3-fold fabric F called the mad weave is shown in Figure 1. Its design $D(F)$ is shown in Figure 2, given by a 3-coloring of the tiling by triangles. The colors yellow, blue and red represent the rankings (123), (231) and (312) respectively, where the three directions of the strands are represented with vectors at 60° with each other, with labels 1, 2 and 3. is shown in Figure 2. The ranking (123) for example would mean a strand with direction 1 goes over a strand with direction 2, which goes over a strand with direction 3.

The layer group representing the symmetry group $S(F)$ of F is given by (G_1, H_1), where each element in G_1 will correspond to a symmetry of $S(F)$ that either preserves or interchanges the sides of F. The elements in G_1 that correspond to a symmetry of $S(F)$ that preserve the sides of F constitute the group H_1, which is of index 1 or 2 in G_1.

For the mad weave we have $S(F) := (G_1, H_1)$ where $G_1 \cong \langle a, b, x, y \rangle \cong p6mm$, $H_1 = \langle a, x, y \rangle \cong p6$; a is the 60° counterclockwise rotation centered at the point labeled P, b is the horizontal reflection passing through P and x, y are translations with vectors indicated. The group $\langle a, b, x, y \rangle \cong p6mm$ is the color group of $D(F)$ and consists of all the elements of the symmetry group of the uncolored triangle tiling that effects a permutation of the colors. On the fabric F, there corresponds is a 60° counterclockwise rotation with center at P' and translations with vectors indicated, that preserve the sides of F, and a reflection whose axis is the horizontal line through P' that reverses its sides.

This paper will discuss all possible layer groups of a 3-way 3-fold isonemal fabric, and give corresponding designs of the fabrics arrived at using color symmetry theory.

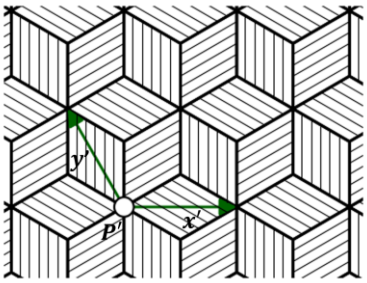

Figure 1. The sketch of the *mad weave*.

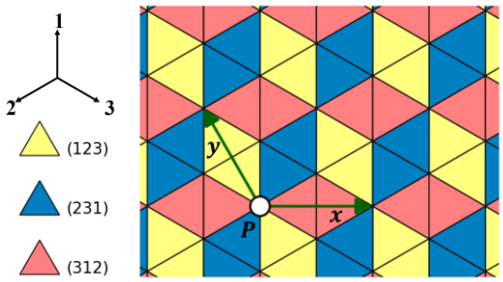

Figure 2. The design of the *mad weave*.

[1] B. Grünbaum, B., Shephard,G. C. (1998). *Isonemal Fabrics.* The American Mathematical Monthly **95**, pp. 5-30.

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