Analysis of Hasse Diagrams of Crystallographic Point Groups Determines Surprising Crystal System Relationships

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This paper is a visual approach to crystallography using Hasse diagrams of the crystallographic point groups. The procedure has been explored in Foundations of Crystallography (Julian, 2015 chapters 3 and 4). Flack, and Gimmer have used a similar approach in their analysis of the Bravais lattices (Flack, 2015; Grimmer, 2015). The result in this paper is a new understanding of the relationships among the point group systems. As a result of symmetry considerations, the 32 three-dimensional point groups are partitioned into those of odd symmetry and those of even symmetry. Also, the orthorhombic and monoclinic point groups are combined into a single group, the dyads. The trigonals and hexagonals are separately partitioned. The point groups of odd symmetry are the triclinics, the trigonals, and the cubics; the point groups of even symmetry are the dyads (monoclinic and orthorhombics), the tretagonals, and the hexagonals. The same analysis is applied to the 10 two-dimensional point groups which are also partitioned into odd and even symmetries. The four crystal systems of oblique, rectangular, square, and hexagonal are examined and rearranged to make them consistent with the three-dimensional case introduced first. The abstract group theory implications are explored. The results are extraordinary considering the simplicity and directness of this approach.

Reference
{1} Flack, H. D. 2015 Acta Crysta. A71m 141-142. Grimmer, M. 2015. Acta Crysta. A71m 143-149
{2] Julian, M. 2015 Foundations of Crystallography with Computer Applications, second edition, Tayler and Francis, Boca Raton