Perfect precise colorings of plane semiregular tilings

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Aside from their aesthetic and mathematical appeal, symmetrically colored tilings have been studied because of their applications in crystallography and materials science. Of particular interest are perfect colorings of tilings, that is, colorings where every symmetry of the uncolored tiling sends all tiles of a given color to tiles of the same color [1, 2]. In addition, colorings of patterns in the hyperbolic plane have garnered attention because of their connection with quasicrystals and structural chemistry.

The term precise coloring was coined by Rigby in [3] to refer to a coloring of the regular triangular tiling (3\( ^7 \)) in the hyperbolic plane in which no two tiles of the same color share a common vertex.

![Figure 1](image)

**Figure 1.** A perfect precise coloring of the tiling (3.3.3.3.9) using 5 colors.

This research is a continuation of work on identifying perfect precise colorings of planar tilings in [4, 5]. We demonstrate how to obtain perfect precise colorings with \( k \) colors of some families of plane semiregular \( k \)-valent tilings where \( k \leq 6 \). These colorings were generated using a combinatorial approach and are verified to be perfect through group-theoretic reasoning. Most of the Archimedean tilings considered by Crowe in [4] fall under the more general class of semiregular tilings examined in this work, and his findings become special cases of results in this paper.