Electrons phase reconstruction using Kramers-Kronig relations

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Transmission electron microscopy (TEM) has been applied as one of the most important techniques to study the local structures of materials with ultra-high resolution. However, a great challenge in TEM is to extract the phase information of electron exit wave, which couldn’t be directly recorded by electron detectors. Why phase information is so important for structure identification in TEM? A simple reason is that the beam energy is much higher than the atomic Coulomb potential. A single-atom interaction leads to a pure phase modulation on the incident wave. Multiple scattering within thick samples may modify the final intensity distributions of electron exit wave, but the phase information is still very critical for crystallographic structures extraction from the complicated interreference intensity patterns, either in real or in reciprocal space. Numerous phase retrieval methods have been developed in TEM and scanning transmission electron microscopy (STEM) for this purpose, including off-axis holography, transport-intensity-equation (TIE) and electron ptychography. These methods are based on the analytical or iterative solution to the wave propagation equations. In this presentation, we introduce a novel phase reconstruction approach based on Kramers-Kronig relations, which has been originally developed in optical imaging [1, 2]. Under KK relations, the real and imaginary parts of a complex function can be transferred into each other when it is analytical in the upper half-plane, leading to a clear connection between the amplitude and phase of the exit wave. We will demo the phase reconstruction for off-axis electron holographic imaging based on KK relations, and make a comparison to the traditional Fourier transform reconstruction as well as least-square method in real space.