Poster

Symmetries and pseudo-symmetries in a sophisticated piece of graphic art and beyond

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Ever since the plane symmetry groups were first derived in 1891 [1], pieces of graphic art (and electron density maps) have been classified with respect to their plane symmetries in a subjective manner. These classifications relied on the visual impression of human classifiers concerning the (necessarily broken) symmetries in what crystallographers like to refer to as finite crystal patterns. Because impressions are subjective and all symmetries in real world objects (including electron density maps) are always broken, not all human classifiers would agree with each other when analyzing the same crystal pattern. If there are very strong pseudo-symmetries, see Fig. 1a, in addition to genuine symmetries, it is unlikely that a scientific consensus might be achieved on the basis of *subjective* symmetry classifications. (Ancient Germanic law has the concept of trial by combat to settle disputes, but the plane symmetry groups were not known before that practice was (according to Wikipedia) abolished by an act of the British parliament in the year 1819).

Whenever a digital version of a finite crystal pattern is available, disagreements about symmetry classifications can be amicably resolved by means of the application of the *objective* methods that this author pioneered in recent years [2, 3]. The plane symmetry of the crystal pattern in Fig. 1a is objectively only *p4*, both per numerical analysis and as direct result of how that particular piece of graphic art has been created! Fellow crystallographers will note that there are in Fig. 1a approximate mirror and glide lines in mutual orientations to apparently combine with four- and two-fold rotation points to plane symmetry group *p4gm*. This author (wrongfully) made this particular plane symmetry assignment himself at first sight, but the sound statistic of a geometric form of information theory [4] convinced him otherwise such that he felt compelled to inquire how this particular piece of graphic art was physically created.

Whenever multiple techniques with inherently different geometric accuracies are combined in the creation of a single piece of graphic art (such as Fig. 1a from a digital copy of a single painted tile, Fig. 1b), one is able—by digital analysis involving the information in all image pixels—to make clear distinctions between symmetries and pseudo-symmetries. The original painting, Fig. 1b, is on a mass-produced ceramic plate that possesses a square shape to a very good approximation. The geometric accuracy that was imposed on that plate's shape by its industrial creation process is far superior to that of its paint coloring, which was added by the artist's hand. Note that there is a broken mirror line along one of the diagonals in Fig. 1b, which results in a pseudo-symmetry in Fig. 1a. The painted tile in Fig. 1b comprises the asymmetric unit in Fig. 1a, i.e. it represents one quarter of the crystal pattern's unit cell as the mirror line is broken to a significantly larger extent than the two-fold and four-fold rotation points that result from the translation-periodic assembly of the whole piece of art. The multiplicity of the general position is therefore *four*, and only two-fold and four-fold rotation points can be considered to be genuine in Fig. 1a, as they arose from the assembly of multiple copies of a digital photograph of the painted tile.

The "take-home" message of this abstract is that crystallographic symmetry classifications of experimental data should better be done *objectively* from now on. If Fig. 1a were representing some electron density map in two dimensions, its crystallographic processing in plane symmetry group *p4gm* would result in mistaking strong pseudo-symmetries for genuine symmetries. That would necessarily [3] result in a "job not well done" and prospectively in the submission of a sub-optimal/incorrect structure to a crystallographic database.

The original length scale of this gray-scale-level rendering of Eva Knoll's `Tiles with quasi-ellipses' (published on page 189 of "M. C. Escher's Legacy", 2003, DOI: 10.1007/3-540-28849-X) remains unknown to this author.

The mass-produced ceramic plate that Eva Knoll used in the creation of her original painted tile has a length of 6 inches (15.24 cm) on each side [2]. A digital photo of that tile was significantly reduced in size for the assembly of her `Tiles with quasi-ellipses' so that the breaking of the diagonal mirror line is almost impossible to notice on sight in sub-figure (a).

Figure 1. Reproduction of a finite crystal pattern by the artist/mathematician Eva Knoll (**a**-left) and its asymmetric unit (**b**-right) [2].

[1] Fedorov, E. (1891). *Proc. Imperial St. Petersburg Mineralogical Society*, *series 2*, **28**, 345 (in Russian).
[2] Moeck, P. (2022). *Acta Cryst.* **A78**, 172 & updated (2023). *arXiv:2108.00829*, 41 pages.
[3] Moeck, P. (2018). *Symmetry* **10**, 133.
[4] Kanatani, K. (2005). *Statistical Optimization for Geometric Computation: Theory and Practice*, 3rd ed., Mineola: Dover Pub. in Mathematics.