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Visual Group Theory: A Computer-Oriented Geometric Introduction. By Stephan Rosebrock. Springer Berlin, Heidelberg, 2024. Price 49.99 Euro. ISBN 9783662693643.

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This book offers a novel approach to studying group theory. The book is part of the Springer Undergraduate Mathematics Series, popularly called the SUMS. There is a greater emphasis on the use of geometry rather than axiomatization and abstraction to understand group theory.

Unlike most other books on the same subject, this book starts with an introduction to basic or fundamental Euclidean geometry and isometries. This is followed by the development of the very important group-theoretic concepts such as subgroups, homomorphisms, group actions, and the classification of finite groups. The author is skilled at providing inspiring and revealing examples for comprehending the subjects. This way, he brings home to readers the essentials of group theory via the use of symmetry and geometric transformations that include rotations, translations and reflections.

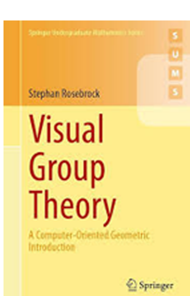
The book is divided into a dozen chapters and around 250 pages. The early chapters are meant to be easily understood, requiring only rudimentary function theory and geometry expertise. Because of this, the material is perfect for self-learners or undergraduates who might not have a strong foundation in mathematics. Later chapters, however, include group presentations, hyperbolic groups and Cayley graphs, and the content is much more complex. This steady increase in complexity guarantees that readers with greater experience can still benefit from the material.

This book stands out for its focus on computational and visual techniques. *GAP* (*Groups, Algorithms, Programming*), a potent open-source software program for computational group theory, is widely used by Rosebrock. *GAP* commands are incorporated within the text to provide interactive visualization and experimentation with group operations and characteristics. This is an excellent touch that captures how mathematical education is changing in the digital age, when computational tools are crucial for expanding knowledge.

The book's strongest point is its focus on geometric representations of groups. The author sends out, very strongly, the message that group theory is better learnt and taught using geometric illustrations rather than abstractions and axioms. Rosebrock uses symmetries in geometric objects like polygons, polyhedra and more sophisticated structures like hyperbolic planes to explain important group-theoretic ideas.

In mathematics, this geometric perspective has a long and illustrious history that dates back to Felix Klein's 'Erlangen Program', which aimed to categorize geometries according to their symmetry groups. Rosebrock continues this tradition by demonstrating how the changes in space can be used to understand group theory. For instance, the dihedral group D_n, which denotes the symmetries of a regular n-gon, is presented as a concrete set of rotations and reflections in addition to an abstract collection of elements. For students who might have trouble with abstraction at an early stage, this method makes the content much more interesting.

The book also relates geometric events to classical group theory discoveries like the Sylow theorems and the categorization of finite groups. Rosebrock is careful to present these findings gradually and logically, providing tangible instances prior to presenting formal proofs. This methodological strategy reflects the way that students often absorb new information best – by initially coming across examples that help them understand the subject matter before being exposed to abstract generalizations.



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In several of the examples in the book, students are urged to use *GAP* to do computations that would be time-consuming or impracticable to complete by hand. By letting readers observe abstract ideas in action, this computational method demystifies them. In the *GAP* environment, for example, students can investigate group actions, determine group ordering, and even categorize small finite groups.

Using *GAP* as a teaching tool is also in line with current developments in mathematics education, where curricula are increasingly including computing. It reflects the fact that there are substantial computational components in many fields of current mathematical research. As a result, Rosebrock's book equips students for both the computational needs of contemporary mathematics and future theoretical study.

Crystallography is one of the main fields where this book can be especially useful. The idea of symmetry groups is fundamental to crystallography, the study of crystal formations and their symmetries. Group theory, especially space groups and point groups, can explain the highly symmetrical patterns that are frequently formed by the arrangement of atoms in crystals.

Group theory is used by crystallographers to categorize crystals based on their symmetries, and the methods described in *Visual Group Theory* – particularly the handling of isometries and finite groups – are immediately useful. Since it clearly explains how symmetry operations work in both two- and three-dimensional environments, the book is a great resource for crystallographers due to its emphasis on the visual and geometric aspects of group theory.

Visual Group Theory by Stephan Rosebrock is a superb addition to the field of teaching mathematics at the undergraduate level. Learning group theory is seen from a new angle thanks to its creative geometric approach and computational tools. The book is easy to read, interesting, and suitable for students with different levels of mathematical ability. Its emphasis on symmetry and visual perception makes it a particularly useful tool for anyone working in practical subjects where symmetry is crucial, like crystallography.

Interestingly, there already exists a book with the same title. It is Nathan Carter's famous book, *Visual Group Theory*, and was published by the Mathematical Association of America in 2009. The books do not just have the same title but they also both aim to make group theory understandable with concerted efforts to focus on intuitive comprehension and visualization. With over 300 illustrations to help readers visualize symmetry and structure, Carter's method mainly focuses on Cayley diagrams and finite groups. His software *Group Explorer* facilitates an engaging educational experience. Rosebrock uses symmetry and visuals in a similar way, but his book incorporates computational tools like *GAP* more thoroughly, enabling real-time group property investigation. Rosebrock's book provides a more comprehensive computational viewpoint, equipping students for both theoretical insights and contemporary computational applications, whereas Carter's work is best suited for novices looking for practical visual experimentation. Both are great resources, but Rosebrock's offers a more sophisticated, computationally based experience, while Carter's concentrates more on basic learning with finite groups.

The book will give readers a thorough understanding of the function that groups perform in both pure and applied mathematics thanks to its blend of geometric and computational content.