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Periodic Platonic solids: the royal family of periodic nets

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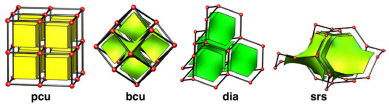
Among all the infinitely many 3D periodic networks you could draw in space, which ones are the most symmetric, in a strict, mathematically defined sense? This is the lattice analog of a very old question: if Platonic solids are the most symmetric finite polyhedra, what plays the same role for repeating structures in 3D?

The pursuit of symmetry is deeply written into how we perceive the world. We instinctively read *healthy* from a round fruit, *robust* from a balanced leaf and *stable* from an evenly branched tree. And the Platonic solids are an ancient manifestation of that instinct: out of all possible polyhedra, only five are perfectly regular, and they have long been treated as a royal family of 3D shapes (Weyl, 2015).

Nets are the periodic cousins of those solids. In crystal and reticular chemistry, a net is a periodic graph that specifies how building units connect in a framework, independent of the atomic details of nodes and linkers. Metal–organic frameworks (MOFs), covalent organic frameworks (COFs) and related materials are routinely described and designed in terms of their underlying nets, a perspective that was highlighted again by the 2025 Nobel Prize in Chemistry recognizing MOFs and reticular chemistry (Yaghi *et al.*, 2003). The natural question then follows: among all 3D nets, which are the most symmetric ones, in the same sense that Platonic solids are among polyhedra?

In the previous issue of *Acta Crystallographica Section A*, Li, O'Keeffe and Treacy propose a precise answer (Li *et al.*, 2025). Building on decades of work on periodic nets, they identify and classify a small and surprisingly exclusive set of 'supersymmetric' 2- and 3-periodic structures in Euclidean space. Their work does not merely introduce yet another family of exotic frameworks. Instead, it completes the portrait of a periodic 'royal family' that has been emerging in the background of reticular chemistry for more than 20 years.

As early as 2005, Delgado-Friedrichs, O'Keeffe, Treacy, Yaghi and co-workers demonstrated that despite the infinite possibilities, many important crystal structures settle into a handful of preferred 3-periodic nets (Delgado-Friedrichs *et al.*, 2005). Their work provided the first global map of this landscape, organizing structures by symmetry, transitivity and tilings. Concurrently, the establishment of the Reticular Chemistry Structure Resource (RCSR) created a vital infrastructure: a curated database with a unified symbol system that quickly became the standard reference for framework design. Subsequent research has linked these nets to natural tilings and triply periodic surfaces, reinforcing the profound insight



that the vast universe of possible nets conceals a surprisingly orderly structure (Delgado-Friedrichs *et al.*, 2005).

In recent years, O'Keeffe and Treacy have advanced this program towards a fully graph-theoretic understanding of periodic structures. In *Embeddings of Graphs: Tessellate and Decussate Structures*, they addressed the fundamental problem of how a given graph maps onto 3D space (O'Keeffe & Treacy, 2024). They distinguished between embeddings that partition space into polyhedral tiles (tessellate) and those that are inherently woven or cross-linked (decussate), arguing that a graph possesses at most one 'optimal' tessellating embedding. Furthermore, the authors and their collaborators have welcomed periodic knots, polycatenanes and weavings into the net family, extending reticular chemistry into the complex realm of entangled structures (O'Keeffe & Treacy, 2025*a*; O'Keeffe & Treacy, 2025*b*; Liu *et al.*, 2018). Collectively, these advances have achieved something rare: they have drawn a precise boundary around the geometry of periodic networks and, crucially, made that boundary practically useful for chemists.

In this context, this new article by Li, O'Keeffe and Treacy represents the 'top shelf' of a carefully curated library. Here, they introduce a stringent definition of supersymmetry for periodic nets. The authors introduce a rigorous criterion for supersymmetry based on the relationship between local connectivity and symmetry. For a given edge-transitive net, they compare the coordination number (n) with the order (o) of the site symmetry group, defining the symmetry number as $s = o/n$. 'Supersymmetric' nets are those rare vertex- and edge-transitive structures where $s > 1$. Intuitively, the local point symmetry at each vertex is not just adequate to permute the bonds, it is more than enough (Li *et al.*, 2025).

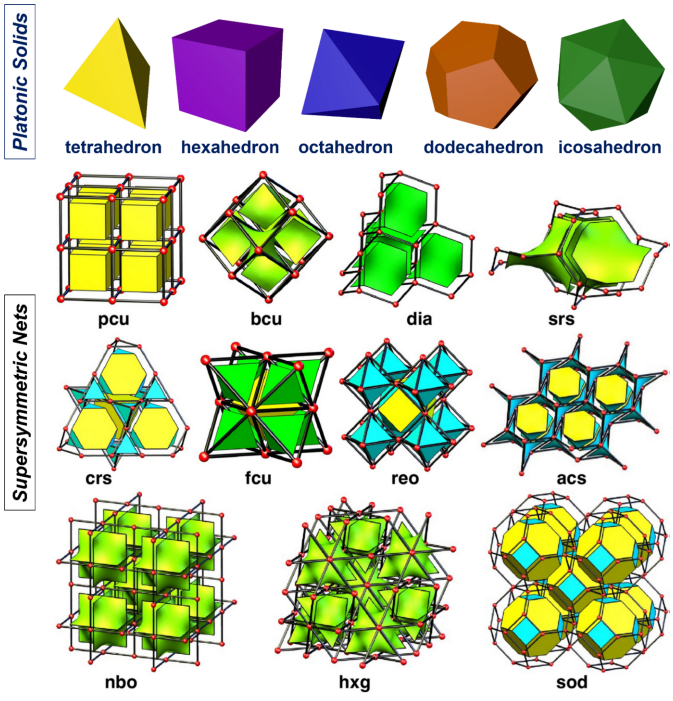


Figure 1
A comparison of high-symmetry finite and periodic structures. The five Platonic solids (top) are shown alongside the 11 connected supersymmetric nets (bottom) classified by Li *et al.* (2025). Both classes of structure are defined by having a symmetry number $s > 1$ (where the site symmetry order exceeds the coordination number).

Once this definition is fixed, the authors tackle the classification problem. They show that there are only 11 connected 3D nets that are both vertex- and edge-transitive and satisfy the supersymmetry condition (Fig. 1). These include several familiar nets that already recur throughout RCSR and the MOF literature, along with a few less obvious frameworks. The authors then relax vertex-transitivity to allow nets with two vertex types but still a single, symmetry-equivalent edge type. Within this transitivity [2 1] class they identify 32 supersymmetric nets. Finally, they extend the discussion to interpenetrating and interwoven versions of these nets, including periodic weavings and polycatenanes constructed from supersymmetric backbones. The small size of these lists is crucial: it reveals that once edge-transitivity and the condition $s > 1$ are imposed, supersymmetric nets are extraordinarily rare in 3D, much like the five Platonic solids among polyhedra.

The implication of this work extends far beyond the traditional domain of porous chemistry. Recent research has increasingly treated framework materials as mechanical systems. From negative compressibility and auxeticity to pressure-induced breathing, these behaviors are often attributed to the deformation modes of the underlying net. Wang and co-workers, for example, have recently constructed 'molecular truss lattices' by choosing MOFs whose topology and connectivity lend themselves to a mechanical metamaterial interpretation, successfully blending reticular chemistry with truss mechanics (Wang *et al.*, 2025). At the macroscopic scale, this geometric logic governs 3D polycatenated architected materials (PAMs), where crystalline nets are translated into interlocked particles that flow, jam and adapt like chainmail (Zhou *et al.*, 2025). In these systems, the net is encoded not in chemical bonds but in mechanical contacts; yet, the same graph topology dictates how the material stiffens, yields and recovers.

Supersymmetric nets sit naturally inside this interdisciplinary domain that runs from molecular frameworks to centimetre-scale lattices. Because they treat all edges and, in the simplest case, all vertices equally by symmetry, they offer a rigorous starting point for studying how symmetry breaking, defects or hierarchy alter material behavior. By including supersymmetric weavings and polycatenanes in their taxonomy, Li, O'Keeffe and Treacy have strengthened the bridge between classical crystallography and the emerging mechanics of tangled matter (Li *et al.*, 2025).

What, finally, is the broader message? One answer is retrospective: this work completes a conceptual map – from the early question 'what do we know about 3-periodic nets?' (Delgado-Friedrichs *et al.*, 2005) to the present identification of supersymmetric nets, O'Keeffe, Treacy and their collaborators have charted how structure, symmetry and topology constrain periodic graphs in 3D. Another answer is more forward-looking. We are entering an era in which, for many

classes of materials, synthesis is increasingly a solvable problem with enough craftsmanship, automation and time. The bottleneck is shifting from 'can we make this structure?' to 'which structure should we make, and why?'. That, fundamentally, is a question about geometry and symmetry.

Foundational classifications like the supersymmetric nets of Li, O'Keeffe and Treacy provide exactly the kind of structured design space that future computational and experimental tools can exploit. They will inform inverse design algorithms, guide mechanical and transport optimization, and help connect disciplines that already think in terms of graphs embedded in space, from soft matter and photonics to robotics and network science. In that sense, the supersymmetric nets identified here are not only beautiful abstractions – they are the Platonic solids of periodic graphs, and they will likely serve as reference shapes not just for chemists, but for materials scientists, engineers and many others who seek to design intelligent matter from the ground up.

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