Supporting information for article:

Tolerance factor and phase stability of the garnet structure

Zhen Song, Dandan Zhou and Quanlin Liu
Tolerance Factor and Phase Stability of the Garnet Structure

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Figure S1: Geometrical relationships used to express the Tolerance Factor

Table S1: Tolerance Factor of End-Member Garnets

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End of Table

References


(40) Arlt, T.; Armbruster, T.; Miletich, R.; Ulmer, P.; Peters, T. High Pressure Single-


(43) Piccinelli, F.; Speghini, A.; Mariotto, G.; Bovo, L.; Bettinelli, M. Visible Luminescence of Lanthanide Ions in Ca$_3$Sc$_2$Si$_3$O$_{12}$ and Ca$_3$Y$_2$Si$_3$O$_{12}$. *J. Rare Earths* 2009, 27, 555–559.


(57) Fuess, H.; Bassi, G.; Bonnet, M.; Delapalme, A. Neutron Scattering Length of Terbium


(59) Malysa, B.; Meijerink, A.; Jüstel, T. Temperature Dependent Cr$^{3+}$ Photoluminescence in Garnets of the Type $\text{X}_3\text{Sc}_2\text{Ga}_3\text{O}_{12}$ ($\text{X} = \text{Lu, Y, Gd, La}$). *J. Lumin.* **2018**, *202*, 523–531.


Tolerance factors of over 130 different end-member garnets together with references, illustration of geometrical relationships used to express the tolerance factor are included in the supporting information.