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A Novel Insertion Device for Circularly Polarized Radiation

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A novel insertion device for circularly polarized radiation with a wide available energy range is proposed. It consists of two helical undulators with different period lengths. From an analysis made on the radiation, higher harmonics are found to be contained in the radiation, unlike the ordinary helical undulator. In addition, calculations show that a high degree of circular polarization is obtained for finite beam emittance.

Keywords: undulators; circular polarization; higher harmonics.

1. Introduction

One of the properties of a helical undulator is that no higher harmonics are observed on-axis (Alferov, Bashmakov & Besonov, 1974; Kincaid, 1977). Although users are never troubled by the heat load caused by higher harmonics, the available photon energy is limited to that of the fundamental radiation, unlike the planar undulator.

In order to obtain higher harmonics with circular polarization, one can choose the so-called crossed undulator (Moissev, Nikitin & Federov, 1978; Kim, 1984). This device contains two linear undulators, one of which is the horizontal undulator and the other is the vertical undulator. When higher harmonics radiated from each undulator are superposed in a certain phase, circular polarization is obtained. However, the degree of circular polarization is degraded easily by the finite beam emittance.

Another way to obtain higher harmonics with circular polarization is the use of an elliptical undulator (Bessonov & Gaskevich, 1985; Yamamoto & Kitamura, 1987). In this case, the amplitude of the vertical motion of the electron is not equal to that of the horizontal motion, therefore the deflection angle, *i.e.* the angle between the electron motion and the undulator axis, is not constant and higher harmonics are obtained. However, it is easily understood that the degree of circular polarization degrades as the difference between the amplitudes of the vertical and horizontal motions becomes larger. On the other hand, the intensity of higher harmonics degrades when the difference is reduced to achieve a high degree of polarization.

In this paper, a novel insertion device for circularly polarized radiation is proposed. It contains two helical undulators (A and B), one of which has a period length three times as long as that of the other. It is shown that quite a wide range of energies will be covered by using this device in three modes of operation, *i.e.* helical undulator A with a period length of λ_u, helical undulator B with a period length of $\lambda_u/3$, and a novel undulator having a composite field obtained from both undulators.

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2. Principles

Let us consider an insertion device containing two kinds of helical undulator with period lengths of λ_u and $\lambda_u/3$, as shown in Fig. 1. The upper array (helical undulator A) has a period length of λ_u and the lower array (helical undulator B) of $\lambda_u/3$. This insertion device has the interesting characteristic that higher harmonics are observed on-axis when both helical undulators A and B are used. This is explained as follows.

First, let us consider the reason why only the fundamental radiation is observed on-axis in the case of the helical undulator. Fig. 2 shows the electron trajectory of the ordinary helical undulator. In this case, the deflection

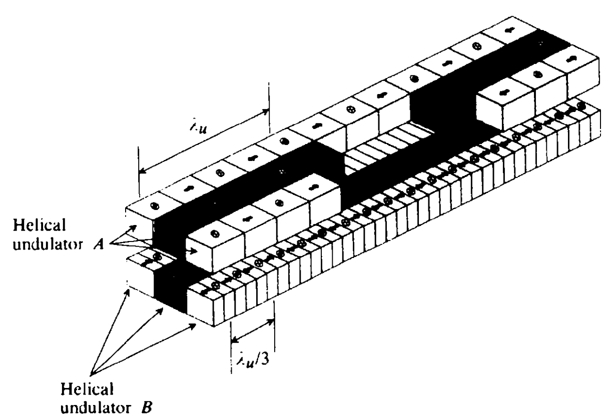

Figure 1
Schematic illustration of the proposed device.

angle, δ, is calculated as

$$\delta = (K/\gamma)(\cos^2 z + \sin^2 z)^{1/2} = K/\gamma, \quad (1)$$

where K is the deflection parameter and γ is the Lorentz factor. Because the deflection angle is constant, the electric field of radiation observed on-axis is never distorted. Therefore, only the fundamental radiation is observed on-axis.

Let us consider a trajectory as shown in Fig. 3. In this case, the deflection angle is equal to zero at the point indicated by the arrows in the figure, where the contribution to the on-axis radiation is the largest. Therefore, the electric field of radiation is distorted and higher harmonics appear in the spectrum. In order to make electrons move along the trajectory shown in Fig. 3, the magnetic fields should be

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1, \quad (2)$$

$$\mathbf{B}_0 = B_0[\cos(2\pi z/\lambda_u), -\sin(2\pi z/\lambda_u), 0], \quad (3)$$

$$\mathbf{B}_1 = B_1[\cos(6\pi z/\lambda_u), \sin(6\pi z/\lambda_u), 0], \quad (4)$$

$$B_1 = 3B_0. \quad (5)$$

From the above equations, it is found that $\mathbf{B}_0$ represents the magnetic field of a helical undulator having a period length of λ_u, while $\mathbf{B}_1$ represents that having a period length of $\lambda_u/3$. Therefore, $\mathbf{B}$ can be generated by adjusting the half gap of each magnet array of the device shown in Fig. 1. Summarizing the discussions above, the proposed device has three modes of operation, *i.e.* two helical undulator modes and the novel undulator mode. In the case of the helical undulator mode, pure radiation without any higher harmonics can be obtained. In the case of the novel undulator mode, higher harmonics with circular polarization can be used for higher photon energies. Let us call the novel undulator mode the rhombus mode, because the orbit in the novel undulator projected on the transverse (xy) plane resembles a rhombus.

Although the magnetic fields shown by equations (2)–(5) are similar to those proposed by Alekseev & Bessonov (1984) for an insertion device for variable polarization, the phase relation between the horizontal and vertical fields is different. Our proposed device contains both right-handed and left-handed helical fields, while their device contains only the right-handed (or left-handed) helical field.

Now let us calculate the spectrum obtained from the novel undulator. Solving the equations of motion we obtain the position and the relative velocity as

$$\begin{aligned}\mathbf{r}(t) = \; & c\beta\{(K/\gamma\omega_0)[(-\sin 3\omega_0 t)/3 + \sin\omega_0 t], \\ & (K/\gamma\omega_0)[(\cos 3\omega_0 t)/3 + \cos\omega_0 t], \\ & (1 - K^2/\gamma^2)t + (K^2/4\omega_0\gamma^2)\sin 4\omega_0 t\},\end{aligned} \quad (6)$$

$$\begin{aligned}\beta(t) = \; & \beta[(K/\gamma)(-\cos 3\omega_0 t + \cos\omega_0 t), \\ & -(K/\gamma)(\sin 3\omega_0 t + \sin\omega_0 t), \\ & (1 - K^2/\gamma^2) + (K^2/\gamma^2)\cos 4\omega_0 t],\end{aligned} \quad (7)$$

with

$$K = eB_0\lambda_u/2\pi mc, \quad (8)$$

$$\omega_0 = 2\pi\beta c(1 - K^2/\gamma^2)/\lambda_u. \quad (9)$$

The spectral intensity for the kth harmonic is calculated as

$$\mathrm{d}^2P_k/\mathrm{d}\Omega\mathrm{d}\Omega = (e^2\gamma^2N^2/\pi\varepsilon_0 c)(|f_x|^2 + |f_y|^2), \quad (10)$$

with

$$f_x = \xi(S_z\gamma\theta\cos\varphi - KS_x)P_N, \quad (11)$$

$$f_y = \xi(S_z\gamma\theta\sin\varphi - KS_y)P_N, \quad (12)$$

$$P_N = (\sin\pi N\omega/\omega_1)/[\pi N(k - \omega/\omega_1)], \quad (13)$$

$$\omega_1 = (4\pi c\gamma^2/\lambda_u)/[1 + 2K^2 + (\gamma\theta)^2], \quad (14)$$

$$S_x = (1/2\pi)\int_0^{2\pi}(-\cos 3\eta + \cos\eta)\exp(-i\psi)\,\mathrm{d}\eta, \quad (15)$$

$$S_y = (1/2\pi)\int_0^{2\pi}(\sin 3\eta + \sin\eta)\exp(-i\psi)\,\mathrm{d}\eta, \quad (16)$$

$$S_z = (1/2\pi)\int_0^{2\pi}\exp(-i\psi)\,\mathrm{d}\eta, \quad (17)$$

$$\psi = k\eta - A\sin 4\eta + B[\sin(3\eta - \varphi)/3 - \sin(\eta + \varphi)], \quad (18)$$

$$A = \xi K^2, \quad (19)$$

$$B = 2K\xi\gamma\theta, \quad (20)$$

$$\xi = k/[1 + 2K^2 + (\gamma\theta)^2]. \quad (21)$$

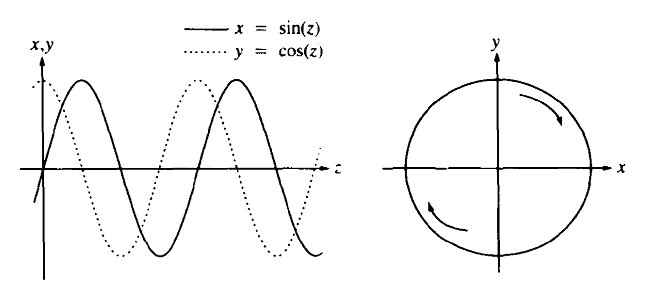

Figure 2
Orbit in an ordinary helical undulator. The deflection angle is always constant.

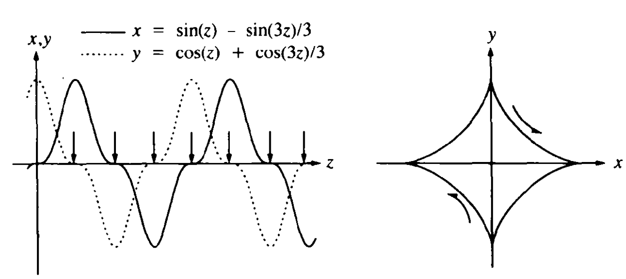

Figure 3
Orbit in the novel undulator. The arrows in the left figure show the points where the deflection angle is equal to zero.

The degree of circular polarization, P_C, is calculated as

$$P_C = [2\,\mathrm{Im}\,(f_x{}^*f_y)]/(|f_x|^2 + |f_y|^2). \tag{22}$$

When observed on-axis, (15) and (16) are rewritten as

$$S_x = \begin{cases} 0; k = \text{even}, \\ -Q; k = 1 + 4n, \\ Q; k = 3 + 4n, \end{cases} \tag{23}$$

$$S_y = \begin{cases} 0; k = \text{even}, \\ iQ; k = \text{odd}, \end{cases} \tag{24}$$

$$Q = (2/\pi) \int_0^{\pi/2} \left[(\sin 3\eta + \sin \eta) \sin (k\eta - A \sin 4\eta)\, \mathrm{d}\eta \right]. \tag{25}$$

It is found from the above equations that P_C is equal to -1 for $(4n + 1)$th harmonics and equal to 1 for $(4n + 3)$th harmonics. In other words, the radiation has both right-handed and left-handed circular polarization. Therefore, we can say that this device cannot be used as a wiggler for circularly polarized radiation. The radiation from the wiggler contains many higher harmonics. The energy difference of the adjacent harmonics is so small that the component of circularly polarized radiation may be easily cancelled out with the electron beam having finite emittance or finite energy spread.

3. Analysis of polarization

In the previous section, we have derived equations on radiation from the novel undulator and found that right-handed and left-handed circular polarization are contained. In this section, we analyze the angular distribution of polarization.

Figs. 4(*a*) and 4(*b*) show the dependence of polarization on the normalized observation angle, $\gamma\theta$, for various harmonics. Each polarization is shown as an ellipse with an arrow representing the direction of rotation. As described before, the on-axis polarization for the $(4n + 1)$th harmonic is right-handed, while that for the $(4n + 3)$th harmonic is left-handed. As for the fundamental, the shape of polarization is found to be almost a circle at all observation angles. On the other hand, the shape of polarization for higher harmonics collapses rapidly with the increase of $\gamma\theta$. After collapsing, the shape begins to return to a circle; however, the direction of rotation is reversed.

Figs. 5(*a*) and 5(*b*) show the angular distribution of the degree of circular polarization, P_C, for harmonics corresponding to those in Figs. 4(*a*) and 4(*b*), respectively. It is found from the figure that the degradation of P_C due to the increase of $\gamma\theta$ is more significant in the case of the higher harmonics. Therefore, P_C for higher harmonics is affected in some degree by the finite beam emittance, which is not so serious for third-generation sources with very low emittance.

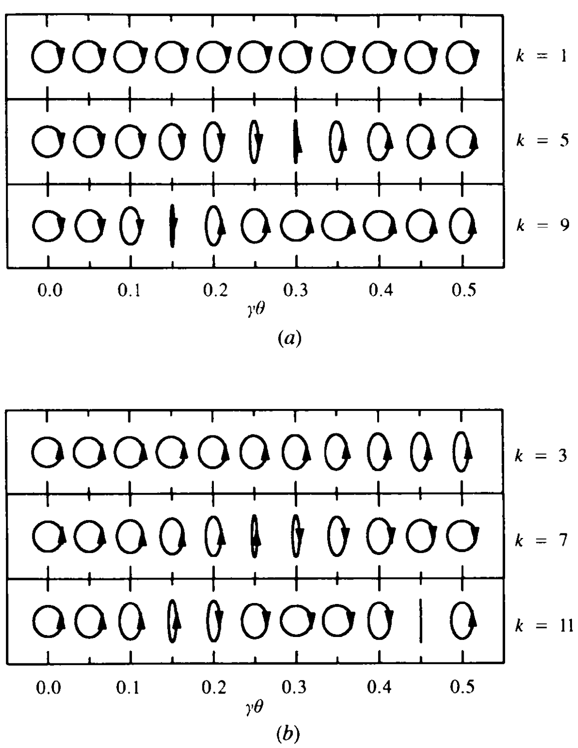

Figure 4
Angular distribution of polarization obtained from the novel undulator for the (*a*) $(4n + 1)$th and (*b*) $(4n + 3)$th harmonics. The arrows in each ellipse show the direction of rotation.

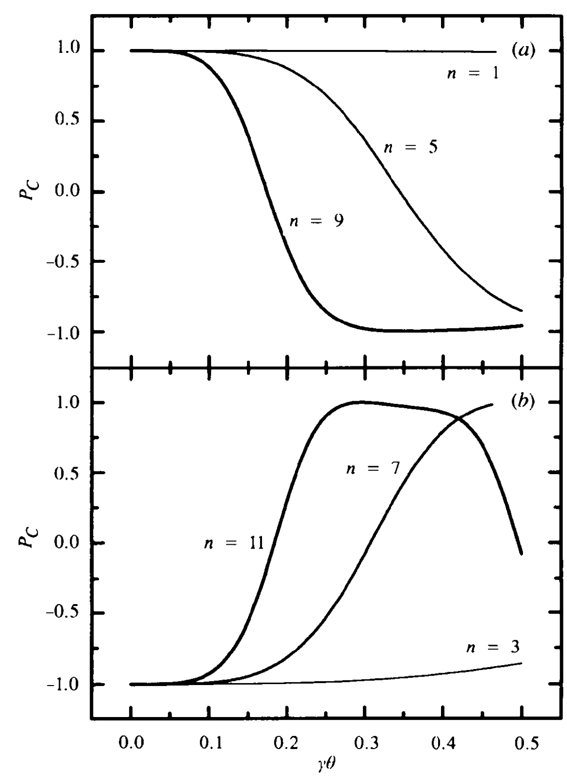

Figure 5
Angular distribution of the degree of circular polarization, P_C, for the (*a*) $(4n + 1)$th and (*b*) $(4n + 3)$th harmonics.

4. Examples

Now let us show some examples of the performance of the novel undulator. The parameters used in the calculation are shown in Table 1. As an example, the period length is assumed to be 18 cm. In this case, the maximum peak magnetic field of the helical undulator B is calculated as 0.18 T at the minimum half gap of 10 mm, which corresponds to a K value of 1.0.

Table 1
Parameters used in the calculation.

Electron energy	1.5 GeV
Beam current	100 mA
Coupling constant	0.1
Horizontal betatron value	10 m
Vertical betatron value	10 m
Total length of insertion device	4.5 m

4.1. *Spectrum*

Figs. 6(*a*)–6(*d*) show examples of spectra obtained from the helical, elliptical, crossed and novel undulators, respectively. The period length in each case is assumed to be 6 cm. The K values for each device are set to the values indicated in the figure so that the energy of the seventh (helical, elliptical and crossed) and the 27th (novel) harmonics may become equal to 1200 eV. The natural emittance is assumed to be 5 nm rad, which is a typical value in third-generation synchrotron radiation facilities. The ratio, K_x/K_y, of the elliptical undulator is set to 0.3 so that the figure of merit, M, represented by $M = P_C(\text{brilliance})^{1/2}$, becomes largest.

It is found from the figure that the helical undulator is the best choice for users who require low-energy photons, because only the fundamental radiation appears in the spectrum. On the other hand, for those who require high-energy photons, *e.g.* above 500 eV, other insertion devices should be used.

Figs. 7(*a*) and 7(*b*) show the spectra and the degree of circular polarization obtained from the elliptical, crossed and novel undulators in the energy range between 1180 and 1220 eV, respectively. If users are not concerned with the degree of polarization, the best choice may be the crossed undulator because the brilliance is the highest. On the other hand, the best choice may be the novel undulator if users need a high degree of polarization.

4.2. *Emittance dependence*

Fig. 8 shows the peak figure of merit and P_C obtained from the elliptical, crossed and novel undulators as a function of the natural emittance. The number of the harmonic is fixed at the seventh (elliptical and crossed undulators) and 27th (novel undulator). It is found from the figure that the figures of merit obtained from each device are almost equivalent to each other. Regarding P_C,

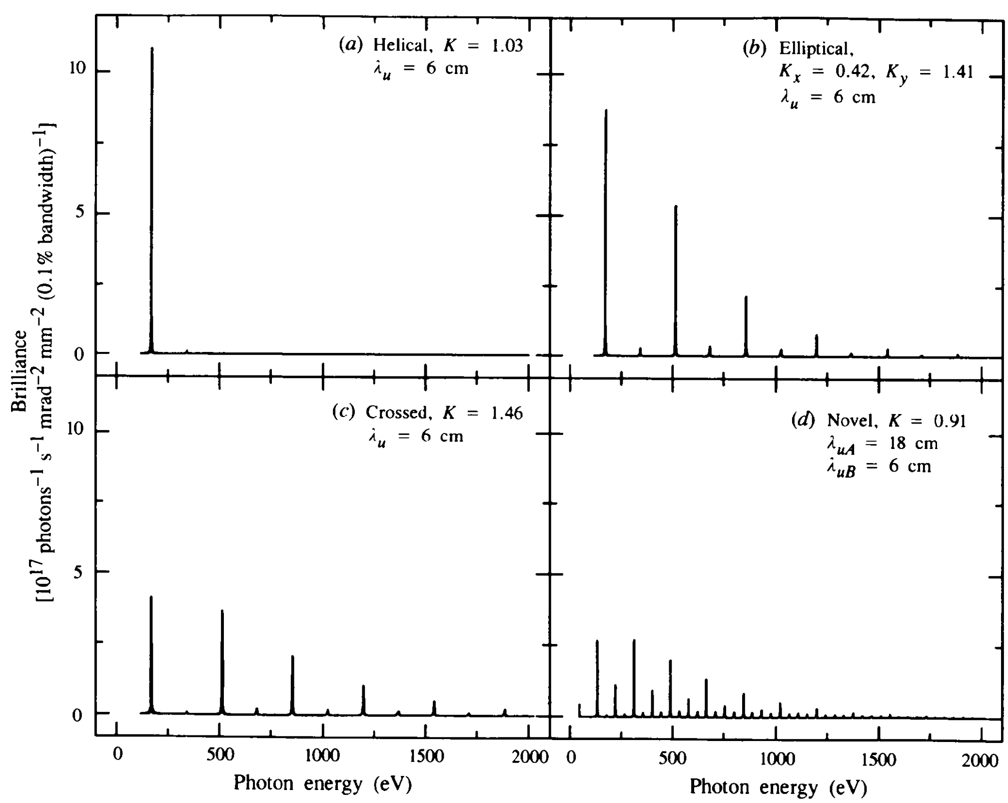

Figure 6
Examples of spectra obtained from the (*a*) helical, (*b*) elliptical, (*c*) crossed and (*d*) novel undulators. The natural emittance is assumed to be 5 nm rad in each case.

the degradation due to the increase of the natural emittance is much more significant in the case of the crossed undulator than in the case of the other two devices. Low P_C means that there are many photons to be abandoned for a user of circular polarization, causing the degradation of the signal-to-noise ratio. We can say that the novel undulator is more useful than the crossed and elliptical undulators when the natural emittance is sufficiently low (< 10 nm rad) because high P_C (~ 1) is available.

4.3. *Available energy range*

Fig. 9 shows the peak brilliance obtained from the proposed device for the three modes of operation. The natural emittance is assumed to be 5 nm rad. The device works as a simple helical undulator when operated in the helical-*A* or helical-*B* mode. The peak brilliance in the rhombus mode is shown only for $(4n + 3)$th higher harmonics because the intensity is higher than those of $(4n + 1)$th harmonics. The dashed line shows an example of the spectrum obtained from an elliptical multipole wiggler (Kitamura & Yamamoto, 1992). The period length of the elliptical multipole wiggler is assumed to be 12 cm, with $K_x = 1.0$ and $K_y = 11.2$. The brilliance obtained from the proposed device is found to be higher than that of the elliptical multipole wiggler in the energy range shown in the figure. We can say from the figure that quite a broad energy range is covered by using the proposed device, as shown in Table 2.

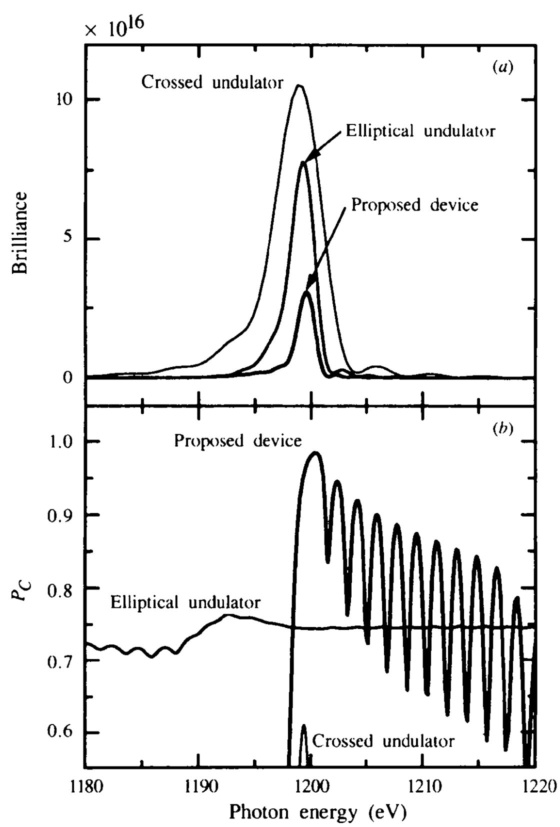

Figure 7
(*a*) Spectra and (*b*) degree of circular polarization, P_C, in the energy range between 1180 and 1220 eV.

5. Summary

The proposed device has been found to have a wide range of the available energy by using three operation modes, *i.e.* the helical-*A*, helical-*B* and rhombus modes. In particular, the rhombus mode is interesting because the radiation contains higher harmonics with circular polarization. Since

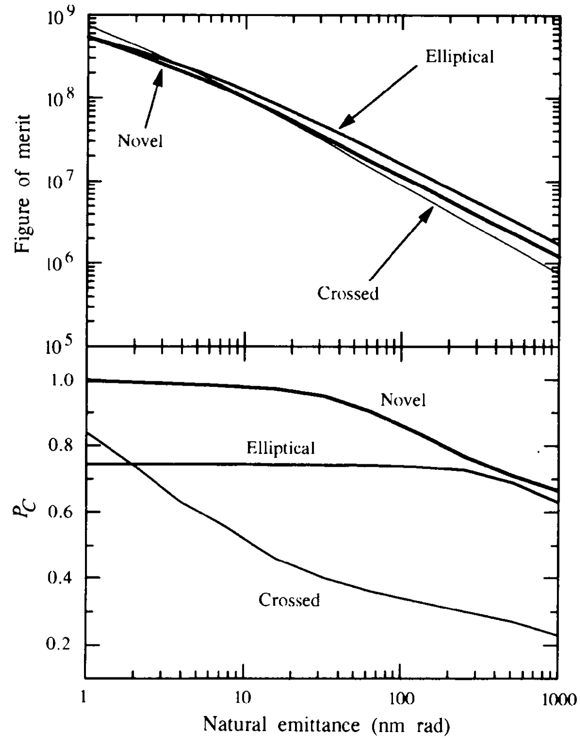

Figure 8
Emittance dependence of figure of merit and P_C for the elliptical, crossed and novel undulators.

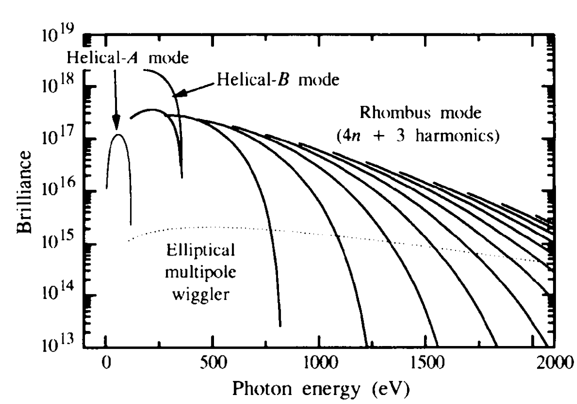

Figure 9
Peak brilliance obtained from the proposed device for three modes of operation and an example of a spectrum obtained from an elliptical multipole wiggler. The natural emittance is assumed to be 5 nm rad in each case.

Table 2
Available energy for the three modes of the proposed device.

Mode	Energy range
Helical-*A*	4–120 eV
Helical-*B*	180–340 eV
Rhombus	120 eV–

the degree of polarization is sufficiently high for finite beam emittance, we can say that this device (rhombus mode) can be used as a circular polarization light source. In order to obtain photons in the vacuum-UV and soft X-ray regions in the synchrotron radiation facilities using medium-energy ($\leq$2 GeV) electrons, it is necessary to use higher harmonics of the undulator radiation. Therefore, we can say that the proposed device is quite valuable for circularly polarized radiation in those synchrotron radiation facilities.

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