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From First-Order Coherence to Higher-Order Coherence of Synchrotron Radiation

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The difference between first-order and second-order coherence of synchrotron radiation is discussed in relation to how they can be measured and how they affect the noise characteristics of future free-electron lasers.

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1. Definition of coherence of light

1.1. *First-order coherence*

First-order coherence is related to whether a light source is regarded to be identical or not. Here, the identity is related to the modes of electromagnetic waves. When emitted light is always confined to a particular single mode, this light has perfect first-order coherence. This coherence has nothing to do with the intensity (Bose degeneracy) of light. Therefore, it is always possible to extract a part of light with 100% first-order coherence using a small collimator by sacrificing the intensity, which is often called van Cittert Zernike's theorem. In fact, observation of light through a collimator or a slit is to project a linear combination of modes into another basis set of modes, because the definition of a mode, and consequently the definition of a basis set of modes, depends on the boundary conditions of the electromagnetic wave, and the existence of a collimator or a slit means a change of boundary condition. The above projection is described by a matrix. The matrix expressing the above projection due to the observation is an example of a density matrix for a pure state in quantum mechanics. For the general case, including a mixed state, the degree of first-order coherence is written as

$$C_1 = Tr[\rho E(\mathbf{r}, t)^* E(\mathbf{r}, t)] / \langle E(\mathbf{r}, t)^* E(\mathbf{r}, t)\rangle, \qquad (1)$$

where the electric field is defined by

$$E(\mathbf{r}, t) = i(h\nu/2\varepsilon_0 V)^{1/2} \sum_j \left\{ \exp[i(\mathbf{k}_j\mathbf{r} - \omega t)]a_j - \exp[-i(\mathbf{k}_j\mathbf{r} - \omega t)]a_j^* \right\}. \qquad (2)$$

a_j and a_j* are the annihilation and the creation operators of a photon with mode j, and ρ is the density matrix. C_1 actually gives the first-order coherence within a certain region of space–time coordinates. When we have a finite region of $\mathbf{r}$ and t, (1) should be integrated in terms of $\mathbf{r}$ and t. In a fermion system a similar expression holds for the first-order coherence, where the many-body wave functions should be used instead of the electric fields because an eigenstate of a fermion system plays the role of a mode of a boson system.

1.2. *Second- and higher-order coherence*

Second-order coherence of light is related to the question of whether two photons have a tendency to come together or to avoid each other. Because a photon is a Bose particle, two photons have a tendency to come together when they are in a thermal equilibrium state described by Bose–Einstein statistics. Here it is convenient to describe the light in terms of a linear combination of photon-number eigenstates such as $|0\rangle$, $|1\rangle$, $|2\rangle$ *etc.* Each photon-number state has a phase oscillation term, $\exp(i\Omega t)$, as the de Broglie oscillation. The phase difference can be defined only relatively. In fact, the relative frequency of the phase oscillations between $|n\rangle$ and $|m\rangle$ states is written as $\Omega = (n - m)\omega$, where ω is the classical frequency of the electromagnetic wave.

It is well known that the appropriate superposition of photon-number states gives the totally coherent state (Glauber, 1963), called the 'Glauber state', where the *expectation value* of the electric field has an exact single frequency, ω, which is identical to the classical frequency of the electromagnetic wave. Under this condition the number of photons observed fluctuates as described by the Poisson distribution. The coherent state is a *pure state* because it is composed of a linear combination of well defined quantum states. It is seen from (1) that the matrix element of the electric field with a single mode does not vanish only between $|n\rangle$ and $|n \pm 1\rangle$ number states, which is the origin of the frequency ω.

For a general superposition of number states the time dependence of the *expectation value* of the electric field is also sinusoidal with frequency ω, because it is still a *pure state*, but the quantum 'fluctuation' of the number of photons observed is more complicated according to the coefficients of the superposition. However, there is also an origin of true (not quantum) fluctuation due to random modulation of the phases of the de Broglie oscillation, which causes a *mixed state*. Therefore, to characterize both fluctuations of number of photons observed, a higher-order observation by correlation measurement of more than two photons is required. The second-order coherence is defined as

$$C_2 = Tr[\rho E(\mathbf{r}_1, t_1)^* E(\mathbf{r}_2, t_2)^* E(\mathbf{r}_2, t_2) E(\mathbf{r}_1, t_1)] / \langle E(\mathbf{r}_1, t_1)^* E(\mathbf{r}_1, t_1)\rangle \langle E(\mathbf{r}_2, t_2)^* E(\mathbf{r}_2, t_2)\rangle. \quad (3)$$

This has the form of the two-photon correlation. We need two photon detectors to measure the second-order coherence, where integration with respect to $\mathbf{r}_1$, t_1, $\mathbf{r}_2$ and t_2 should be performed for a finite detection area of the detectors. Usually the above measurement is performed for spatial separation $X \equiv |\mathbf{r}_1 - \mathbf{r}_2| >> |\mathbf{r}_1|$, and $|\mathbf{r}_2|$, which is the case for the famous experiment by Hanbury-Brown and Twiss. Third- or higher-order coherence can be defined in a similar way as (3). It is to be emphasized that lower-order coherence is a necessary but not sufficient condition of higher-order coherence.

Fig. 1 shows three typical cases of second-order coherence with parameter X, where A, B and C correspond to totally coherent radiation, thermal radiation and an example of squeezed radiation. It is well known that the probability of coincidence of n photons is $n!$ times larger in thermal radiation than in totally coherent radiation such as lasers. It can be seen from Fig. 1 that the probability for thermal radiation with $X = 0$ is twice as large as that of totally coherent radiation. It should be noted that the totally coherent radiation has a flat response for any higher-order coincidence.

A partially squeezed state, such as case C, is a state where two photons avoid being in the same state, like fermions. This state can be achieved only through the interaction of radiation with a fermion system, such as by fluorescence or non-linear absorption. Strictly speaking, it is useless to define the second- or higher-order coherence of a fermion system because a fermion does not have any chance of occupying the same state as another fermion due to Pauli's exclusion principle. Therefore, second- or higher-order coherence could be largely changed when photons interact with a fermion system. There is, however, an apparent exception, in which higher-order coherence seems to be defined even for a fermion system, where quasi-Bosons are formed by combination of fermions.

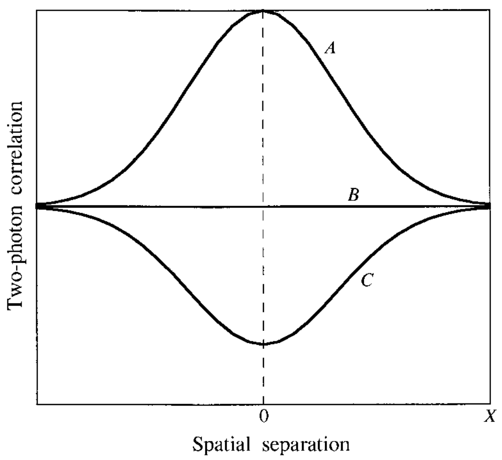

Figure 1
Three typical cases of two-photon correlation. A is the case of thermal radiation, where the peak at $X = 0$ is twice as large as case B, which is for totally coherent radiation. C is the case for a partially squeezed state.

2. Measurement of coherence of synchrotron radiation

2.1. *First-order coherence*

A Young's interferometer was constructed to measure the transverse spatial coherence of synchrotron radiation in the soft X-ray region. The apparatus included a grating monochromator to keep the temporal coherence as good as possible. Using this apparatus, the first-order spatial coherence of synchrotron radiation was measured at BL12A and BL28A of the Photon Factory, KEK. Fig. 2 shows examples of the Young's interference pattern observed, from which the visibility corresponding to the first-order coherence is clearly defined. Because the spacing between the two slits and the wavelength can be changed as parameters, we can estimate the emittance of the stored electron beam by observing the turning point at which the emittance of light is equal to that of the stored

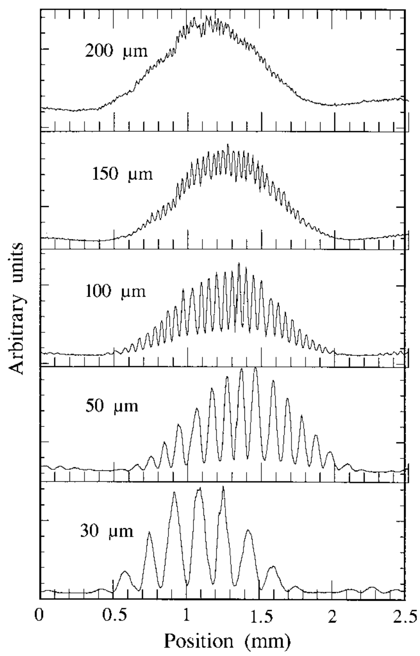

Figure 2
Examples of the Young's interference patterns of synchrotron radiation. The photon energy is fixed at 180 eV, while the spacing of the double slit is changed from 30 to 200 μm.

beam. This is a by-product of the measurement and it turns out that the apparatus has the ability to measure emittances as low as 0.1 nm rad. A detailed description is presented elsewhere in these proceedings (Takayama *et al.*, 1998).

2.2. *Second-order coherence*

The second-order coherence for a stationary light source was first measured by Hanbury-Brown and Twiss by two-photon coincidence (Hanbury-Brown & Twiss, 1956). They adopted a very elaborate method of extracting the two-photon correlation by photoelectric measurement. For synchrotron radiation, however, there is a difficult problem due to the time structure of the stored beam caused by bunch formation. In fact, when we have two photon detectors, and one detector detects photons with a high probability due to the peak intensity of a bunch, then the other detector also has a high probability of detecting photons because the latter also sees the same bunch structure. This is not a true correlation but an accidental or trivial correlation. This is regarded as accidental because it could happen even when there is no first-order coherence.

In order to find a way of extracting only the true correlation we consider the effect of the time structure of the light source. The correlation, including both true and accidental, measured by the correlator electronics with DC cut filters is calculated as

$$\begin{aligned}\langle[N_1(1+f_1)-\langle N_1\rangle][N_2(1+f_2)-\langle N_2\rangle]\rangle\\ &= \langle N_1N_2\rangle(1+f_1+f_2+f_1f_2)-\langle N_1\rangle\langle N_2\rangle(f_1+f_2+1)\\ &= (\langle N_1N_2\rangle-\langle N_1\rangle\langle N_2\rangle)(1+f_1+f_2+f_1f_2)\\ &\quad+\langle N_1\rangle\langle N_2\rangle f_1f_2, \end{aligned}\tag{4}$$

where N_1 and N_2 are the numbers of photons detected by two detectors, D_1 and D_2, respectively, and $\langle\ldots\rangle$ means the statistical average. The time structures, f_1 and f_2, of the stored beam observed by the two detectors are so defined by

$$f_1 = f_1(t), \qquad f_2 = f_2(t), \tag{5}$$

where $\langle f_1\rangle = \langle f_2\rangle = 0$, that the constant terms normalized to unity are subtracted from the time structures in (4). Apparently, f_1 and f_2 are periodic functions of t, at least with a period of the revolution frequency of the stored beam. Actually, f_1 and f_2 have very high frequency components associated with the bunch length.

If the light source is stationary, and thus f_1 and f_2 vanish, it is relatively easy to measure the correlation. However, the difficulty for a non-stationary light source is that the first term of the third line of (4) is much smaller than the second term, though we want to abstract the true correlation C_t included in the first term as

$$C_t \equiv \langle N_1\rangle\langle N_2\rangle(C_2-1) = \langle N_1N_2\rangle - \langle N_1\rangle\langle N_2\rangle. \tag{6}$$

In (6), $C_2 - 1$ is roughly equal to $n_B\tau/T_m$, where n_B is the Bose degeneracy of the light source, τ is the coherence time of a photon determined by the monochromator resolution, and T_m is the response time of the detectors. Usually, τ is much smaller than T_m and n_B is much smaller than unity, which gives $C_2 - 1$ of the order of 10^{-4} in the case of an undulator beamline BL28A of the Photon Factory.

Therefore, we have designed two methods of overcoming the above difficulty. One is to modulate τ by modulating the width of the entrance slit of the monochromator, inducing a change in the spectral resolution. Then we can extract the true correlation using lock-in amplifiers. The other is to use special notch filters in the electronics after the two detectors to suppress the effect of f_1 and f_2 in (4) by sacrificing the signal intensity. If the suppressed Fourier components are 10^{-4} times smaller than the case without the notch filters, then we can regard the light source as a 'stationary' source. A detailed description will be presented elsewhere (Takayama *et al.*, 1998).

3. How the second-order coherence of synchrotron radiation appears

For the general description of the photon statistics, including the squeezed state, a quantum mechanical description of the system is necessary to satisfy the uncertainty principle in measuring photon occupation numbers and their phases. However, there are two extreme cases, totally coherent radiation and thermal radiation, which can be described by classical electromagnetism with a couple of assumptions for photons. The assumptions are as follows.

(i) Each mode of electromagnetic wave acts as a 'vessel' in which any number of identical photons can be included.

(ii) The number of photons is proportional to the square of the amplitude of the electromagnetic wave.

We start by defining the Gaussian wave packet in $\omega - t$ space, where the standard deviations $\Delta\omega$ and Δt satisfy the uncertainty condition, $\Delta\omega\Delta t = 1/2$. This wave packet is regarded as a single mode if the other remaining part in the four-dimensional phase space is in a single mode. Thus, the wave packet is regarded as occupying the minimum volume in six-dimensional phase space. Obviously the above wave packet is *not* a simulation of a photon but a vessel for photons.

First, the radiation emitted by a single electron running on a definite orbit is regarded as totally coherent because the phases of the radiated electromagnetic wave have no ambiguity or statistical fluctuations. This means that when the electromagnetic wave is decomposed into many Gaussian wave packets, their phase relation is uniquely determined. In this situation the two-photon correlation shows a flat response against any of the six parameters defining the phase volume, as indicated by B in Fig. 1. This is just due to random coincidence obtained when a coherent beam is split into two beams on which the correlation is measured. Thus, the time sequence of incoming photons obeys the Poisson distribution and the corresponding fluctuation is often called the 'particle noise'.

Next, consider an ensemble of electrons which 'independently' radiate the totally coherent electromagnetic wave. Here we assume that the phases of the electromagnetic waves radiated by different electrons are random and these electromagnetic waves *interfere* with each other. The important thing is that as a result of interference the photon occupation number, n, can be changed, because n is proportional to the square of the amplitude of the electromagnetic wave. This means that two sets of radiation emitted by two electrons are *not independent* but *correlated*. Suppose two identical wave packets, each having n photons, are separated by a time interval t. When $t << \Delta t$ and these waves interfere positively, the resultant photon number is $4n$, which is twice as large as the original total photon number $2n$. However, when the interference is negative, the photon number almost vanishes. Therefore, when positive and negative interference occurs randomly, it gives 100% fluctuation in the number of photons, which is the main feature of thermal radiation and is often called the 'wave noise'. When τ increases, the interference becomes more and more incomplete and converges to the flat background with $t >> \Delta t$. Consequently, the Poisson-like time sequence of the wave packets emitted by many electrons gives the two-photon correlation as indicated by A in Fig. 1.

The above discussion leads to two important conclusions.

(i) The wave packets described above *never* simulate the corresponding photons because they are *not* quantum mechanical wave functions. On the contrary, when the phases of the wave packets do not fluctuate at all, the photon statistics obey the Poisson distribution, and when the former obeys the Poisson distribution, the latter obeys the Gaussian distribution with a Poisson-like background.

(ii) Noise consideration for future FELs is important when their operation is not perfect. Obviously the 'particle' noise for coherent radiation is equal to $n^{1/2}$ while the 'wave' noise for thermal radiation is n. When we consider the case where spontaneous emission with n photons is amplified with gain g, the 'particle' noise and the 'wave' noise are of the order of $(gn)^{1/2}$ and n, respectively. In order to have the condition where the thermal noise is negligible, we should satisfy the condition $g >> n^{1/2}$, because otherwise radiation due to imperfect FELs would still be noisy compared with totally coherent radiation.

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