Simple scheme for harmonic suppression by undulator segmentation

Takashi Tanaka* and Hideo Kitamura

The Institute of Physical and Chemical Research, Koto 1-1-1, Mikazuki-cho, Sayo-gun, Hyogo 679-5148, Japan. E-mail: ztanaka@spring8.or.jp

In order to suppress harmonic intensity of undulator radiation, a scheme similar to the detuning technique used in the double-crystal monochromator is investigated and found to be effective only when the number of periods of the undulator is small, once the finite emittance of the electron beam and angular acceptance of the beamline are taken into account. Instead, a simple scheme is proposed for undulators with many periods: the undulator is divided into several segments and the optical phase in between is adjusted to shift the fundamental energy without significantly affecting other harmonics.

Keywords: undulator radiation; harmonic suppression.

1. Introduction

It is well known that the spectrum of undulator radiation (UR) has a series of sharp peaks with the width proportional to the reciprocal of the number of periods of the undulator. In the case of a conventional undulator, the peak energies are integer multiples of a certain minimum energy (fundamental energy) determined by the electron energy, magnetic strength and periodic length. It is therefore impossible to obtain pure radiation without contamination by higher harmonics only with a conventional monochromator.

One solution to this problem is the detuning technique generally used in a double-crystal monochromator (Batterman & Bilderback, 1991). Because the Darwin width of the second crystal is narrower for higher harmonics, tilting it slightly from the Bragg angle (detuning) results in a reduction of the higher-harmonic intensity. In the soft X-ray region, however, such a technique cannot be applied. The unwanted higher harmonics are in some cases removed by optical elements dedicated to harmonic rejection, which in turn causes a degradation of the available flux.

Another solution is to adopt the quasi-periodic undulator (QPU) proposed by Hashimoto & Sasaki (1994) and improved by several authors (Chavanne *et al.*, 1998; Sasaki *et al.*, 1998; Diviacco *et al.*, 2000). The spectrum of QPU consists of harmonics with energies which are not integer multiples of the fundamental energy, enabling monochromated radiation to be obtained with much less harmonics than that obtained with a conventional undulator. Several QPUs of pure-permanent magnet and hybrid types have been constructed. The disadvantages of these devices are that they cannot be restored to normal, *i.e.* periodic devices. The electromagnet QPU (Schmidt *et al.*, 2001) can be restored to the periodic device; however, a longer periodic length is in general necessary to construct an electromagnet undulator than others.

In this article we propose simpler schemes to suppress the higher-harmonic intensity. Two different methods are shown. The first one is based on a principle analogous to that of detuning in the double-crystal monochromator and does not require any special optical or magnetic instruments; however, it has a narrow application. On the other hand, the second one, which is the main subject of this article, can be applied in any case.

2. Principle

The principle of the scheme to be proposed is based on detuning. Two methods are presented. One is to detune the photon energy with the photon beam being monochromated, and the other is to divide the undulator into several segments and detuning the optical phase in between.

2.1. Energy detuning

Apart from the term describing the intensity specific to the harmonic number, the spectral profile of the kth harmonic of UR is dominated by the function S defined by

$$S(N,\theta)=\left(\frac{\sin N\theta}{N\theta}\right)^2,$$

with

$$\theta=\pi(k-\omega/\omega_1),$$

where ω is the photon energy, N is the number of periods and ω_1 is the fundamental energy. Let us consider the case where the photon beam is monochromated at the energy $\omega_m=\omega_1+\Delta\omega$. Clearly, the kth harmonic energy selected by the monochromator is equal to $k\omega_m=k(\omega_1+\Delta\omega)$, which leads to the UR intensity proportional to $I_k = S(N,\pi k\Delta\omega/\omega_1)$. If $|\Delta\omega|$ is smaller than $\omega_1/(kN)$, I_k is a decreasing function of $k|\Delta\omega|$. In other words, the bandwidth of each harmonic is proportional to the reciprocal of the harmonic number. Thus, it is possible to reduce the higher-harmonic intensity if we monochromatize the photon beam at an energy slightly shifted from ω_1.

2.2. Phase detuning

As discussed later, the scheme described in the preceding section is not effective in the case where the number of periods is large, once the emittance of the electron beam and angular acceptance of the beamline are taken into account. In this section an alternative method is presented.

Let us consider an undulator composed of M segments with number of periods N and drift spaces in between. In this case the electric field of UR is composed of M wave trains separated by the time interval ΔT. To obtain the spectrum of radiation, the Fourier transform of the electric field is calculated as follows,

$$\begin{aligned}\mathbf{E}_\omega &= \int_0^{NT}\mathbf{E}(t)\exp -i\omega t\,\mathrm{d}t+\int_{\Delta T+NT}^{\Delta T+2NT}\mathbf{E}(t-\Delta T)\exp -i\omega t\,\mathrm{d}t+\ldots\\ &=\sum_{m=0}^{M-1}\int_{mNT}^{(m+1)NT}\mathbf{E}(t)\exp -i\omega(t+m\Delta T)\,\mathrm{d}t\\ &=\sum_{k=1}^{\infty}\mathbf{E}_k\,\frac{1-\exp iM\left[2\pi N(k-\omega/\omega_1)-\omega\Delta T\right]}{1-\exp i\left[2\pi N(k-\omega/\omega_1)-\omega\Delta T\right]}\\ &\quad\times\frac{1-\exp 2\pi iN(k-\omega/\omega_1)}{i(k\omega_1-\omega)},\end{aligned}$$

where $\mathbf{E}_k$ is the kth component of the Fourier series obtained by expanding the electric field $\mathbf{E}$ in one period. Let us consider again the case where the photon beam is monochromated at the energy $\omega_m=\omega_1+\Delta\omega$. The kth harmonic intensity extracted by the monochromator is proportional to I_k, calculated as

$$I_k = \left[\frac{\sin kM(\pi N\Delta\omega/\omega_1 + \Delta\varphi/2)}{M\sin k(\pi N\Delta\omega/\omega_1 + \Delta\varphi/2)}\right]^2 S(N, k\pi\Delta\omega/\omega_1), \qquad (1)$$

where we have introduced a phase-mismatch parameter $\Delta\varphi$ by

$$\Delta\varphi = \omega_1 \Delta T.$$

I_k consists of two terms. The first term denotes the interference effect between segments, whereas the second term describes the normal spectral profile of UR.

Now let us consider the case where $\Delta\varphi = 4\pi/3$, $M = 2$ and $N = 50$. Substituting into equation (1), it is found that I_k for the third harmonic reduces to $S(MN, k\pi\Delta\omega/\omega_1)$, a normal profile of UR with the number of periods of MN, while that for the fundamental is significantly distorted as shown in Fig. 1. The fundamental peak is shifted to higher energy, while that at the third harmonic remains the same. If M is increased with the product MN being constant, the fundamental peak is shifted further. This means that the third-harmonic intensity will be considerably reduced if we choose a large number of M and monochromatize the photon beam at the energy of the shifted fundamental peak.

3. Effect of angular divergence and acceptance

The discussions in the preceding section are made under the condition that the electron beam has zero emittance and energy spread, and the beamline has an infinitely small angular acceptance, the effects of which are investigated in this section. The parameters used in the calculation are summarized in Table 1, the beam parameters of which are typical values for the medium-energy third-generation synchrotron radiation facilities recently constructed or under planning. Energy spectra of the flux passing through a rectangular slit with horizontal and vertical full widths of $4\Sigma_{x,y}$ are calculated, where $\Sigma_{x,y}$ is the root-mean-square photon beam size observed at the slit position. All calculations have been made in the near-field region with a synchrotron radiation calculation code, *SPECTRA* (Tanaka & Kitamura, 2001), developed at SPring-8.

3.1. Energy detuning

Let us first consider the case with the energy detuning. Energy spectra of harmonics up to the fifth are plotted in Fig. 2. In order to clarify the detuning effect, the abscissa is shown with the photon energy normalized by the harmonic number. Two different sets of λ_u and N are considered. When N is equal to 10 (Fig. 2*a*), the energy detuning is effective to suppress the higher-harmonic intensity without sacrificing significantly the fundamental flux. Detuning the photon energy by $\Delta\omega$ = 1.5 eV leads to about 60% degradation of the fundamental flux, while it reduces the intensity of other harmonics by two orders of magnitude. In the case of N = 100, however, the energy detuning is not effective, as shown in Fig. 2(*b*). Owing to the finite emittance and angular acceptance, the peak width is broadened for each harmonic, which spoils the detuning effect. Energy detuning of $\Delta\omega$ = 1.7 (2.6) eV degrades the fundamental intensity to 50 (20)%. Nevertheless, the third harmonic still remains with an intensity of

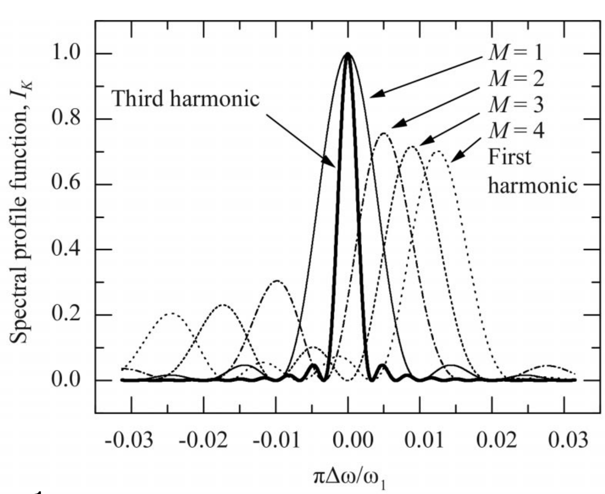

Figure 1
Phase detuning effect caused by the segmentation of the undulator and insertion of mismatched phase.

Table 1
Parameters used in the calculation.

Electron energy	2 GeV
Average current	100 mA
Natural emittance	10 nm rad
Energy spread	0.001
Coupling constant	1%
Horizontal betatron value	24 m
Vertical betatron value	10 m
Undulator total length	5 m
Undulator *K* value	2
Slit distance from the source	30 m

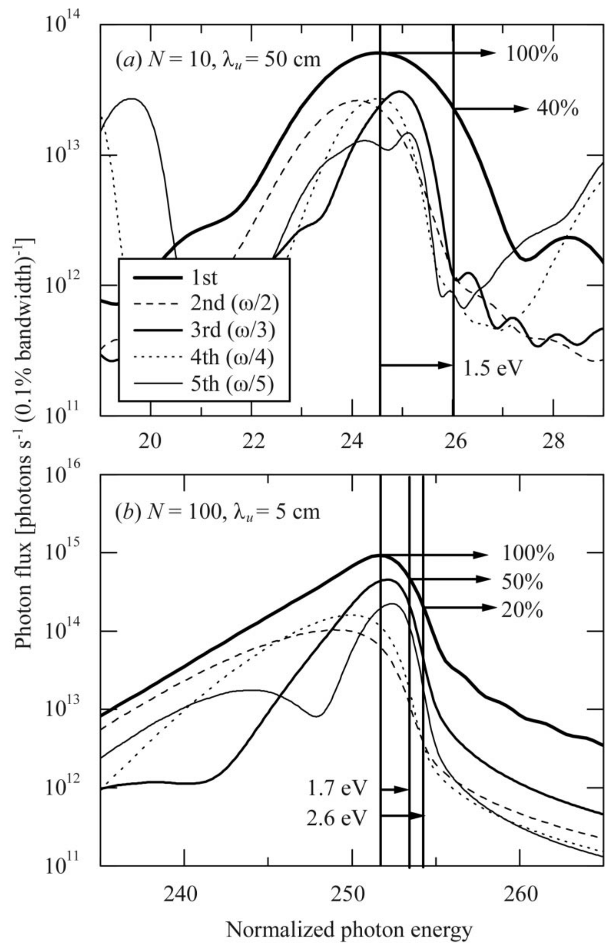

Figure 2
Harmonic suppression by the energy detuning. Energy spectra up to the fifth harmonic considering the finite emittance and angular acceptance are shown for two difference cases: (*a*) N = 10 and λ_u = 50 cm and (*b*) N = 100 and λ_u = 5 cm.

Table 2
Summary of the energy-detuning effects.

	$N = 10$, $\Delta\omega = 1.5$ eV		$N = 100$, $\Delta\omega = 1.7$ eV		$N = 100$, $\Delta\omega = 2.6$ eV	
Harmonic	Flux	Ratio (%)	Flux	Ratio (%)	Flux	Ratio (%)
1st	2.4×10^{13}	100	4.6×10^{14}	100	1.8×10^{14}	100
2nd	1.1×10^{12}	4.6	1.2×10^{13}	3.3	3.3×10^{12}	1.8
3rd	1.1×10^{12}	4.6	2.2×10^{14}	55	3.6×10^{13}	20
4th	8.6×10^{11}	3.6	1.7×10^{13}	3.7	3.1×10^{12}	5.5
5th	8.5×10^{11}	3.6	1.1×10^{14}	24	1.5×10^{13}	8.3

55 (20)% of the fundamental. In Table 2 the harmonic suppression effects by the energy detuning are summarized in both cases of $N = 10$ and 100.

3.2. Phase detuning

Next let us consider the case with the phase detuning. In order to create a phase mismatch $\Delta\varphi$ of $4\pi/3$, a drift space of length $(2/3)\lambda_u(1 + K^2/2)$ has been inserted, where λ_u and K are the periodic length and deflection parameter of the undulator.

Energy spectra up to the fifth harmonic are plotted in Fig. 3 for different numbers of segments. As in Fig. 2, the energy is normalized by the harmonic number. The total number of periods, or the product MN, is assumed to be 100 and the periodic length is 5 cm. The spectra shown in Fig. 3(*a*) for the case of $M = 1$ are therefore the same as those in Fig. 2(*b*). They are shown just for comparison with other cases. The fundamental peak energy is shifted due to the segmentation and phase mismatch, while those of other harmonics remain almost the same. The fundamental degradation is about 30% in both cases. In the case of two segments, the phase detuning may not be sufficient. The flux ratios of the third and fifth harmonics to that of the fundamental are about 35% and 16%, respectively. Increasing the number of segments considerably improves the situation. In the case of four segments, the ratios of all harmonics are less than 2.4%. The phase-detuning effects are summarized in Table 3.

4. Conclusion

We have presented two simple schemes to suppress higher-harmonic intensity of UR. In the case of the energy detuning, no additional instruments are necessary. The user of UR is required to simply shift the energy slightly from the value where the fundamental intensity has its maximum. As to the phase detuning, several mechanical modifications and instruments are necessary. However, the scheme is simple. We simply divide the entire undulator into several segments and detune the phase between them. In §3 we used a simple drift section to create phase mismatch; however, it is more useful and flexible to install a magnetic phasing section to create a bump of the electron orbit to tune the phase. The magnetic field of the phasing section should be variable to ensure the phase mismatch condition $\Delta\varphi = 4\pi/3$ for all the gap values of the undulator. Needless to say, setting $\Delta\varphi = 2\pi$ reproduces a normal undulator with a length of the entire device, which is a great advantage over the QPU.

The phase-mismatch condition $\Delta\varphi = 4\pi/3$ introduced in this article is to reduce the ratio of the third-harmonic intensity to the fundamental. If the user wanted to use the third harmonic and eliminate the ninth, $\Delta\varphi$ of $[2\pi - 2\pi/9 = 16\pi/9]$ would work well. The value of $2\pi/9$ is derived from the ninth harmonic and the minus sign is to obtain the peak shift to higher energy in order to avoid contamination by higher harmonics due to the regular low-energy tails.

Table 3
Summary of the phase-detuning effects.

Note that the flux at the fundamental energy for the case of no segmentation ($M = 1$) is equal to 30×10^{14}.

	$M = 2$		$M = 4$	
Harmonic	Flux	Ratio (%)	Flux	Ratio (%)
1st	6.7×10^{14}	100	6.2×10^{14}	100
2nd	1.7×10^{13}	2.5	1.2×10^{13}	1.9
3rd	2.4×10^{14}	36	1.0×10^{13}	1.6
4th	2.7×10^{13}	4.0	5.4×10^{12}	0.9
5th	1.1×10^{14}	16	1.5×10^{13}	2.4

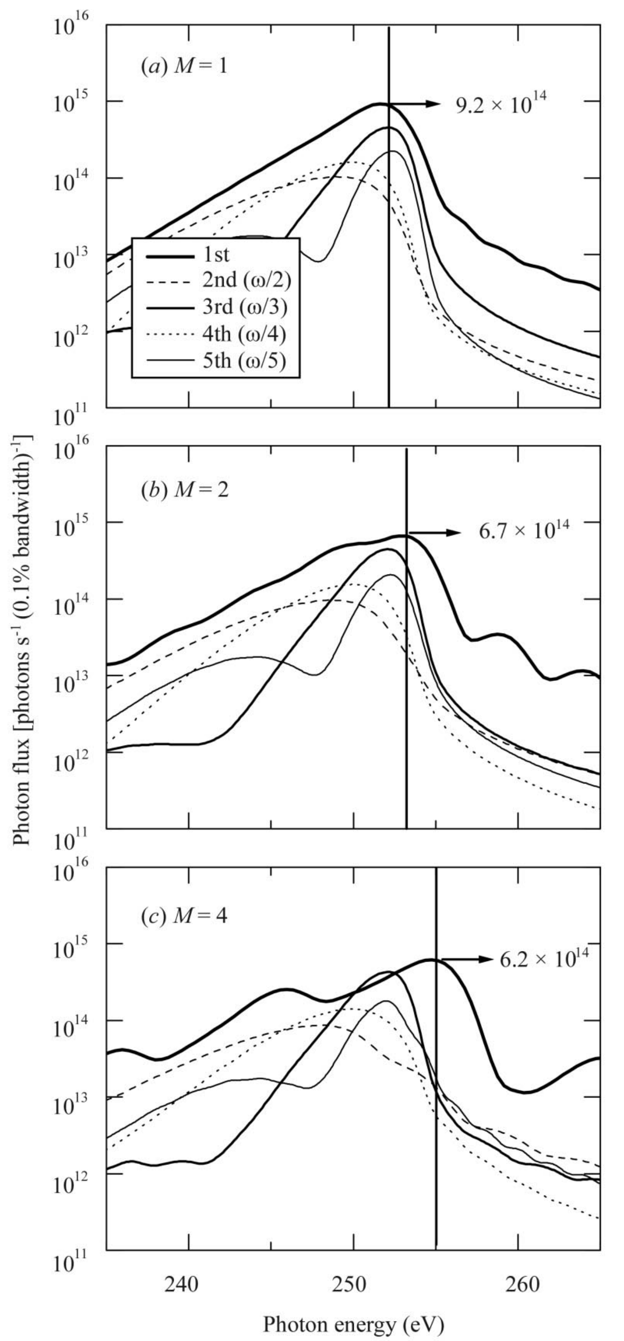

Figure 3
Harmonic suppression by the phase detuning. Energy spectra up to the fifth harmonic are shown for different numbers of segments: (*a*) $M = 1$, (*b*) $M = 2$ and (*c*) $M = 4$.

Only a planar undulator is considered in this article. It is worth noting that the proposed scheme can be applied to other undulators such as the figure-8 (Tanaka & Kitamura, 1995) and elliptic undulators. For the latter, the scheme is exactly the same as that presented so far. In the case of the figure-8 undulator, $\Delta\varphi$ should be optimized because even harmonics also appear in the spectrum which is not the case for standard undulators.

References

Batterman, B. W. & Bilderback, D. H. (1991). *X-ray Monochromators and Mirrors*, ch. 4 in *Handbook on Synchrotron Radiation*, edited by G. S. Brown & D. E. Moncton. Amsterdam: North-Holland.

Chavanne, J., Elleaume, P. & Van Vaerenbergh, P. (1998). *Proceedings of the 6th European Particle Accelerator Conference (EPAC-98)*, Stockholm, Sweden, pp. 2213–2215.

Diviacco, B., Bracco, R., Millo, D., Walder, R. P., Zalateu, M. & Zangrando, D. (2000). *Proceedings of the 7th European Particle Accelerator Conference (EPAC 2000)*, Vienna, Austria, pp. 2322–2324.

Hashimoto, H. & Sasaki, S. (1994). JAERI-M report 94-055. JAERI, Hyogo, Japan.

Sasaki, S., Diviacco, B. & Walker, R. P. (1998). *Proceedings of the 6th European Particle Accelerator Conference (EPAC-98)*, Stockholm, Sweden, pp. 2237–2239.

Schmidt, T., Ingold, G., Imhof, A., Patterson, B. D., Patthey, L., Quitmann, C., Schulze-Briese, C. & Abela, R. (2001). *Nucl. Instrum. Methods*, A**467**/**468**, 126–129.

Tanaka, T. & Kitamura, H. (1995). *Nucl. Instrum. Methods*, A**364**, 368–373.

Tanaka, T. & Kitamura, H. (2001). *J. Synchrotron Rad.* **8**, 1221–1228.

 Received 11 January 2002 • Accepted 25 March 2002