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Feasibility of a low-energy X-ray free-electron laser

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X-ray free-electron lasers based on self-amplified spontaneous emission promise users unprecedented X-radiation that is extremely bright, extremely short and transversely coherent. However, hard X-ray free-electron laser facilities under construction are all huge and expensive, consisting of high-energy linear accelerators and long undulators. The benefit of hard X-ray free-electron lasers may be limited to only a few regions in the world, unless it is possible to reduce the size. This paper discusses how small a hard X-ray free-electron laser facility can be. It is shown that a 1.5 Å X-ray free-electron laser is achievable using electron energy down to 4.5 GeV or lower if we use the third-harmonic radiation, but at the expense of the transverse coherence.

Keywords: X-ray free-electron lasers; self-amplified spontaneous emission; saturation length; coherence.

1. Introduction

A laser system in the X-ray regime has long been a dream in the related science community. Now it is being realised in the form of a free-electron laser (FEL) based on self-amplified spontaneous emission (SASE) (Bonifacio *et al.*, 1984). The FEL is a linac-based light source that uses electron bunches passing through an undulator in combination with a seed laser. The laser–electron interaction causes micro-bunching spaced in the wavelength, which emit hugely magnified coherent radiation. A SASE FEL is a type of FEL that uses its own radiation instead of that from an X-ray seed laser which is not available. There are three outstanding properties of a SASE FEL: extreme brightness, extremely short pulses and transverse coherence.

Apparently an X-ray FEL (XFEL) is achievable only using a high-energy electron beam. To make a 1–1.5 Å hard X-ray FEL, the electron energy has been chosen to be 14.35 GeV for the Linac Coherent Light Source (LCLS) at Stanford Linear Accelerator Center (SLAC) (Arthur *et al.*, 2002) that is under construction, and 17.5 GeV for the European XFEL at DESY (Aghababyan *et al.*, 2006) that has been approved. With XFELs, it is not only the linear accelerator but also the undulator that is long; the LCLS undulator is 112 m long and the European XFEL undulator is even longer at 260 m. We may have to conclude that hard X-ray FELs are too expensive to be available in most countries. The current hard X-ray FEL projects are listed in Table 1. Is it possible to build one at a reasonable cost? The SPring-8 Compact SASE Source (SCSS) project in Japan is attempting to reduce the whole facility size by using an in-vacuum undulator and the new technology of C-band linear accelerators (SCSS XFEL R&D Group, 2005). It is going to need only an 8 GeV electron beam to generate hard X-rays. However, building and maintaining an 8 GeV electron machine still costs a lot, even with the new technology. Is there a possibility of a compact XFEL machine that is affordable to a modest-size laboratory, and, if it is possible, how compact would it be? This is what we want to investigate in this paper. Obviously, arbitrarily low electron energy would not generate XFELs. We will show that it is possible to generate hard X-rays using a 4.5 GeV electron beam, approximately 30% of the LCLS energy but only at the expense of full transverse coherence. Low-energy hard X-ray FELs are useful only for experiments that do not need transverse coherence.

2. Compact XFEL

XFELs consist of a linear accelerator, undulator and beamline. In this paper we will concentrate on downsizing the linear accelerator and undulator. To reduce the accelerator size, we should use a low-energy electron beam. The resonant wavelength of an undulator is given by

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right), \tag{1}$$

Table 1
Hard X-ray FEL projects.

	Wavelength (Å)	Electron energy (GeV)	Undulator period (cm)	Full undulator length (m)
LCLS	1.5	14.35	3	112
European XFEL	1	17.5	3.56	260
SCSS	1	8	1.5	81

where λ_r is the resonant wavelength, λ_u is the undulator period, γ is the Lorentz factor and K is the undulator parameter. K is defined by

$$K = 0.934 B_0\,[\mathrm{T}]\ \lambda_u\,[\mathrm{cm}], \tag{2}$$

where B_0, the undulator peak magnetic field, depends not only on the undulator gap and period but also on the magnet material. For a hybrid undulator with vanadium permendur, B_0 is given by

$$B_0 = 3.694 \exp\left[-5.068 \frac{g}{\lambda_u} + 1.520 \left(\frac{g}{\lambda_u}\right)^2\right], \tag{3}$$

with g denoting the gap. At LCLS, for λ_r = 1.5 Å, the beam energy is 14.35 GeV, λ_u = 3 cm and g = 0.65 cm. If we want to use a lower beam energy we have to use a shorter λ_u and smaller K that depends on λ_u and B_0. Since B_0 depends on g/λ_u, we have two parameters (λ_u and g/λ_u) to be controlled to compensate for the decreasing beam energy. Hence, we fix g/λ_u (and thus B_0) and use only λ_u. Solving (1) for λ_u while keeping the LCLS value of the ratio g/λ_u, *i.e.* 0.217, we can determine the value of λ_u that gives 1.5 Å hard X-rays at a lower electron energy. First, re-arranging (1) for λ_u we obtain a cubic equation,

$$\lambda_u^3 + \frac{2}{a^2}\lambda_u = \frac{4\lambda_r\gamma^2}{a^2}, \tag{4}$$

where $a = 0.934B_0$. Solving this cubic equation, we obtain λ_u as a function of γ or E, the electron energy. A graph of λ_u *versus* E is shown in Fig. 1. As E decreases from the LCLS energy, λ_u decreases almost linearly. Since g/λ_u is fixed, $g = 0.217\lambda_u$ also decreases making an in-vacuum undulator an inevitable choice at lower electron energies.

To build a compact XFEL, we also have to reduce the undulator length. To estimate the necessary undulator length at a lower E, we have to compute the SASE saturation length, L_{sat}, and find its energy dependence. A key parameter for the determination of L_{sat} is the FEL parameter ρ defined by

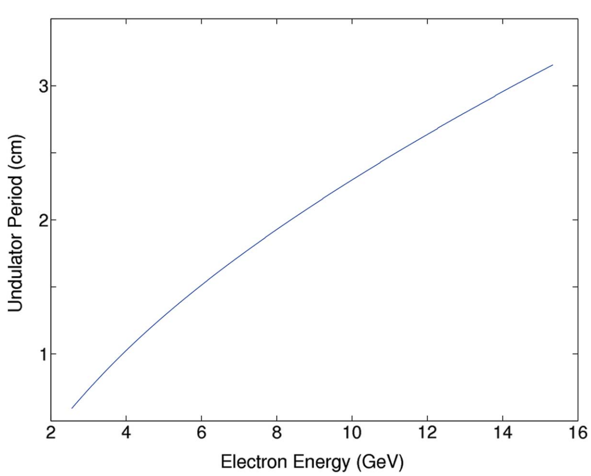

Figure 1
Graph of λ_u that gives 1.5 Å radiation as a function of E. The ratio g/λ_u is fixed to 0.217, the LCLS value.

$$\rho = \frac{1}{2\gamma}\left[\frac{I}{I_A}\frac{\lambda_u^2 K^2 [JJ]^2}{8\pi^2\sigma_x^2}\right]^{1/3}, \tag{5}$$

where I_A = 17045 A is the Alfen current, I is the beam peak current, σ_x is the cross-sectional beam size, and $[JJ]$ is defined by

$$[JJ] = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right). \tag{6}$$

Note that ρ roughly describes the SASE FEL efficiency as in

$$\rho \simeq \frac{\text{generated field energy}}{\text{electrons kinetic energy}}. \tag{7}$$

Hence, a large ρ means high radiation power. ρ also defines the upper bound of the electron energy spread σ_E/E in a slice. The physical space where the FEL process happens is not the whole bunch but each of many slices in a bunch. The SASE process begins only when $\sigma_E/E < \rho$ and it stops (saturates) when σ_E/E grows and reaches ρ. Again, ρ should not be too small for successful power growth. The fundamental length scale to determine the saturation length is the one-dimensional gain length defined by

$$L_{1D} = \frac{\lambda_u}{4\sqrt{3}\pi\rho}. \tag{8}$$

A one-dimensional parameter is what is obtained by a one-dimensional theory that does not include diffraction effects. In general, a large ρ is preferred not only for high gain but also for a short gain length.

In (5), note that $\sigma_x^2 = \beta\varepsilon_n/\gamma$ where ε_n is the normalized emittance and β is the betatron function. The currently achievable value for ε_n is 1.2 µrad and β is free to choose. The optimal β that gives the shortest saturation length is given by (Saldin *et al.*, 2006)

$$\beta_{opt} = 11.2\left(\frac{I_A}{I}\right)^{1/2}\frac{\varepsilon_n^{3/2}\lambda_u^{1/2}}{\lambda_r K\,[JJ]}. \tag{9}$$

Using β_{opt} in (5), we obtain

$$\rho = \frac{1}{2}K[JJ]\left(\frac{I}{I_A}\frac{\lambda_u}{\varepsilon_n}\right)^{1/2}\left(\frac{\lambda_r}{89.6\pi^2\varepsilon_n\gamma^2}\right)^{1/3}. \tag{10}$$

Using the LCLS value I = 3.4 kA, the dependence of ρ on E as λ_u moves on the line of Fig. 1 is shown in Fig. 2. Note that ρ also decreases as E decreases. The requirement $\sigma_E/E < \rho$ gives a severe restriction for a compact XFEL source. The LCLS value of σ_E/E is approximately 0.01%, which means $\sigma_E \simeq$ 1.4 MeV. As the electron energy E is lowered, the relative energy spread σ_E/E increases while ρ decreases. At around E = 4.5 GeV, σ_E/E is comparable with ρ. Hence, E = 4.5 GeV seems to be the lowest possible energy for a 1.5 Å XFEL.

One-dimensional theory is a nice tool to explain the FEL physics. However, it is not accurate numerically. When diffraction effects are not negligible, L_{1D} is not an accurate description of the gain length (Kim, 1986; Yu *et al.*, 1990; Chin *et al.*, 1992). Computation of the corresponding three-dimensional parameter L_{3D} is greatly simplified by the parametrization,

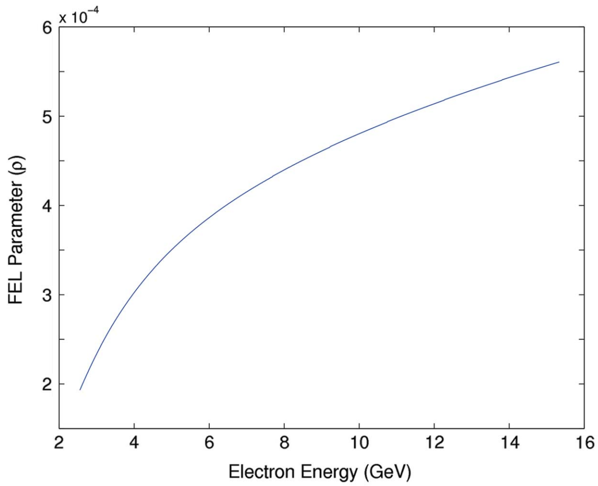

Figure 2
Graph of ρ as a function of E.

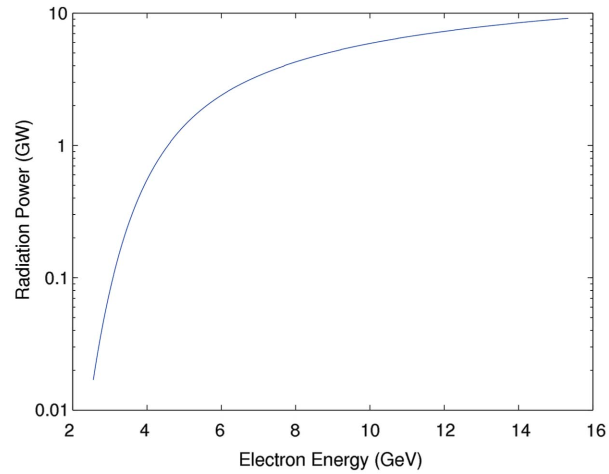

Figure 4
Graph of P_{sat} as a function of E.

$$L_{3D} = L_{1D}(1 + \eta), \tag{11}$$

where η measures the deviation from the one-dimensional theory owing to diffraction, emittance and energy spread. A very useful formula for estimating η was obtained by parameter fitting to simulation results (Xie, 1995). L_{sat} and P_{sat}, the saturated peak radiation power, are approximately given by

$$P_{sat} = 1.6\rho\left(\frac{L_{1D}}{L_{3D}}\right)^2 \frac{I\gamma mc^2}{e},$$
$$L_{sat} = L_{3D} \ln\left(\frac{P_{sat}\lambda_r}{2\rho^2 Ec}\right). \tag{12}$$

Certainly, L_{sat} is an important factor for determining the whole machine size. Using (5) and (11) in (12), the E-dependence of L_{sat} is revealed, and is shown in Fig. 3. L_{sat} also decreases as E decreases from the LCLS energy and reaches the minimum at around E = 5–6 GeV. In Fig. 3, the part below E = 4.5 GeV is meaningless, because the energy spread exceeds ρ and there is no SASE process. The abnormal abrupt increase of the saturation length indicates its meaninglessness. P_{sat} is depicted in Fig. 4 on a logarithmic scale. Note that P_{sat} decreases slowly as E decreases from the LCLS energy to $E \simeq$ 4.5 GeV and drops rapidly outside of it. P_{sat} is still above 1 GW. Therefore, a compact XFEL does not sacrifice the radiation power, let alone the short pulse length. Overall, the shortest XFEL for 1.5 Å can be built at around E = 4.5 GeV.

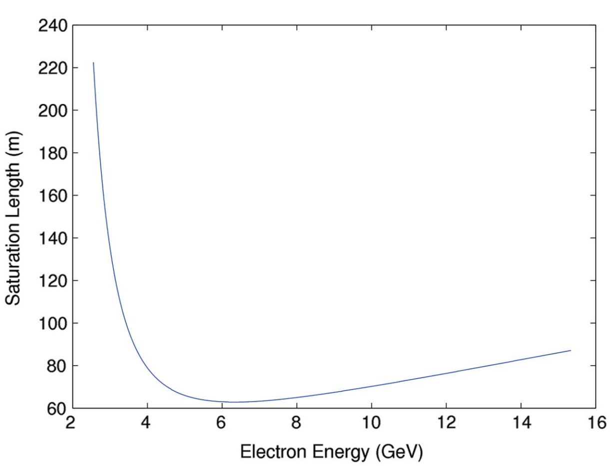

Figure 3
Graph of L_{sat} as a function of E.

3. Issue of the undulator wakefield

To build a low-energy hard X-ray FEL, one has to worry about the undulator wakefield effect, especially the relative energy spread within a bunch induced by the longitudinal resistive wall wakefield, because its effect is a few times larger than the LCLS case. There are two causes for this large effect: small undulator gap and low electron energy. For a 4.5–5 GeV hard X-ray FEL, the undulator gap (practically undulator chamber gap for in-vacuum undulators) is around 2.5 mm, which is only half of the LCLS chamber gap. This small gap causes not only beam handling difficulty but also non-negligible resistive wall wakefield. The large energy spread induced by the small undulator gap is magnified by the low electron energy when we consider the relative energy spread, which is given by

$$\delta_E = -e^2 NL\langle W_z\rangle/E, \tag{13}$$

where $\langle W_z\rangle$ is the average wake function over a bunch and L is the undulator length. Evaluation of $\langle W_z\rangle$ depends not only on the chamber geometry but also on the bunch shape, but in general it is inversely proportional to the undulator gap. The relative energy spread for a 4.5–5 GeV XFEL is around 4.5 times larger than the LCLS value, unless the chamber geometry is different. Including both DC and AC conductivity for the wakefield calculation (Bane & Stupakov, 2004), δ_E is estimated approximately to 0.9% (assuming an aluminium-coated flat chamber), which is much larger than ρ. It should be kept to a few times of ρ. Since it spreads energy between slices, not within a slice, it does not prevent the FEL process from occurring but causes slices with large energy deviation to radiate out of resonance. The final result would be simply the radiation power reduction. To minimize the reduction, it is recommendable to reduce δ_E by using a lower charge (smaller N). Since lowering the charge also lowers the radiation power, one should be careful to choose an optimal charge to obtain a net gain and maximal power. Also, note that a low charge configuration can give better performance to XFELs (Emma *et al.*, 2005). In any case, power reduction is unavoidable for a low-energy hard X-ray FEL. However, the reduction is never serious; it is not an order of magnitude reduction. The radiation power is still huge.

4. Issue of the transverse coherence

There is another requirement for an XFEL machine, the transverse coherence. The transverse phase space of the SASE radiation consists of many spatial modes in the early stage. Among the many spatial modes, only the fundamental mode is centered and Gaussian shaped, while other higher modes are larger-sized and have stronger diffraction properties. As an electron bunch passes through the undulator, higher modes diffract fast and do not accumulate. Only the fundamental mode accumulates and grows to be the only remaining mode. This is how the transverse coherence is achieved in SASE and is called the mode selection (Moore, 1985; Scharlemann *et al.*, 1985).

The condition for transverse coherence is roughly given by

$$\varepsilon_n/\gamma \simeq \lambda_r/4\pi. \qquad (14)$$

This rough condition claims that the beam energy has to be high enough to secure the transverse coherence for a very small λ_r (hard X-ray). Since (14) is an order-of-magnitude relation, accurate estimate of transverse coherence requires a computer. The degree of transverse coherence at saturation was obtained as a function of $\hat{\varepsilon} = 2\pi\varepsilon_n/(\lambda_r\gamma)$ (Saldin *et al.*, 2006). Converting this result for our purpose, we obtain Fig. 5, which shows clearly that the degree of transverse coherence for 1.5 Å hard X-rays decreases as the electron energy decreases. According to Fig. 5, the degree of transverse coherence of LCLS is approximately 0.83. Fig. 6 shows the corresponding radiation beam profile of LCLS at the end of

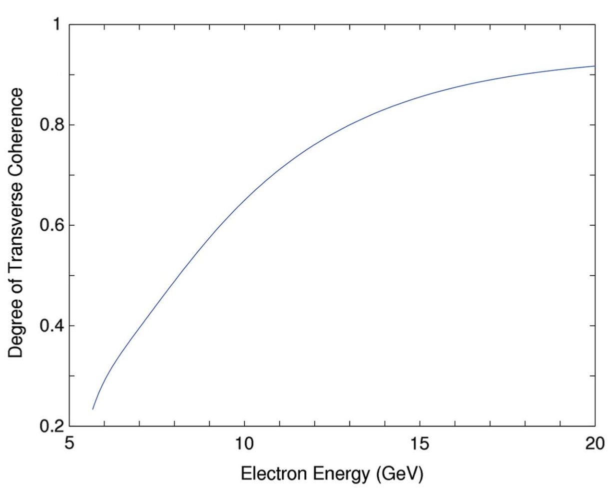

Figure 5
Degree of transverse coherence for a 1.5 Å XFEL as a function of E.

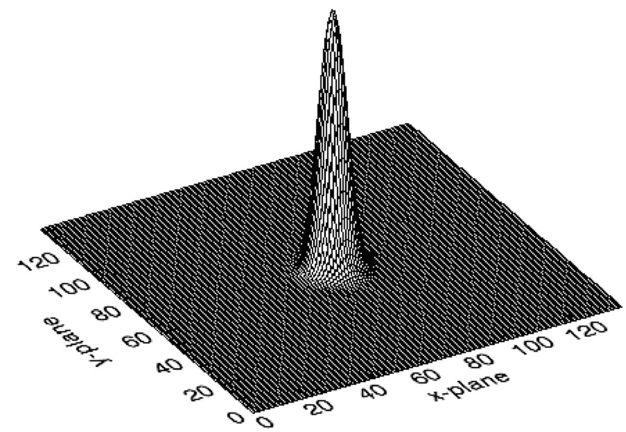

Figure 6
Radiation profile of LCLS at the end of the saturation length (breaks included).

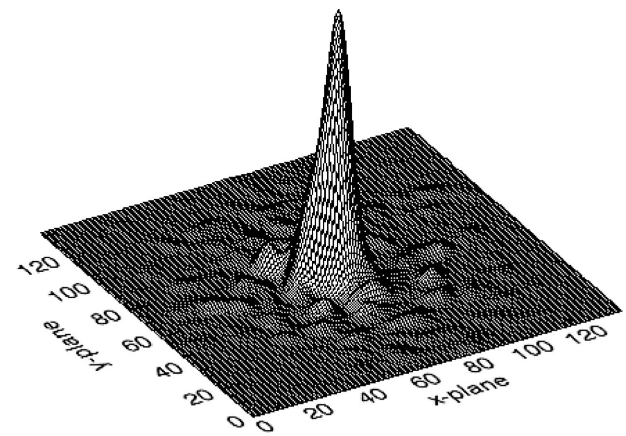

Figure 7
Radiation profile of a low-energy XFEL with E = 5.95 GeV and λ_u = 1.5 cm. The position is again the end of the saturation length. Still, the fundamental mode is the highest, but other minor modes clearly exist.

the saturation length given by (12). This was simulated using the *Genesis* code (Reiche, 1999). We see that it consists of the central and Gaussian fundamental mode and is transversely coherent enough. As an example, if we choose the combination of λ_u = 1.5 cm and E = 5.95 GeV from Fig. 1, its degree of transverse coherence would be low, around 0.3. Its simulation shows incomplete transverse coherence as shown in Fig. 7. Still, the fundamental mode is the highest, but other modes are shown to exist, although they are lower. At a lower energy and shorter undulator period the transverse coherence would be worse. Therefore, we conclude that a hard X-ray FEL is achievable at a lower electron energy but its transverse coherence may not be perfect.

5. Use of higher harmonics

An even smaller hard X-ray FEL facility is possible if we use the third harmonic of SASE radiation. In SASE, coherent harmonic radiation is emitted by the so-called non-linear harmonic generation (Colson, 1981; Bonifacio *et al.*, 1990; Huang & Kim, 2000). If we generate 4.5 Å fundamental

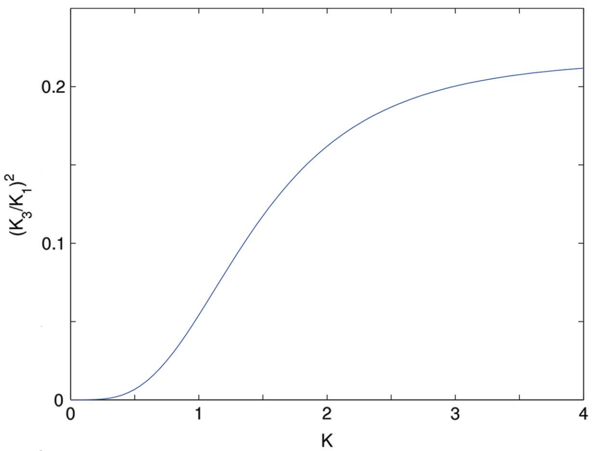

Figure 8
Graph of $(K_3/K_1)^2$ as a function of K.

radiation at an even lower energy, its 1.5 Å third-harmonic radiation is usable. The output power of the third harmonic is much lower than that of the fundamental mode. The ratio of the third harmonic power to the fundamental power is given by (Saldin *et al.*, 2005)

$$P_3/P_1 = 0.094(K_3/K_1)^2. \quad (15)$$

K_1 and K_3, the coupling factor of the fundamental and the third harmonic, respectively, are special cases of K_h, defined by

$$K_h = K(-1)^{(h-1)/2}\left[J_{(h-1)/2}(Q) - J_{(h+1)/2}(Q)\right], \quad (16)$$

where $Q = hK^2/(4 + 2K^2)$. It is straightforward to compute $(K_3/K_1)^2$ as a function of K. As shown in Fig. 8, it increases from zero and becomes almost flat after $K > 2.5$, saturating to $(K_3/K_1)^2 = 0.22$, which gives the asymptotic value $P_3/P_1 \simeq 0.02$. Hence, P_3 cannot exceed 2% of P_1. The degree of transverse coherence of the third-harmonic radiation, obtained at low energy, is also low.

Therefore, a hard X-ray FEL facility can be even smaller if the third-harmonic radiation power (1–2% of the fundamental radiation power) is high enough. In many cases it is still huge. The proposed PAL-XFEL project is supposed to give 3 Å of fundamental and 1 Å of third-harmonic radiation with 3.7 GeV electrons and estimated saturation length of 45 m (Lee *et al.*, 2006). With $K = 1.49$, P_3 is approximately 1% of P_1.

6. Conclusion

The storage-ring-based third-generation light source has spread all over the world in the last 20 years and is now a useful and common facility for scientific research. However, an even more advanced X-ray source, the XFEL facility, is not likely to be so. The LCLS consists of a long linear accelerator of 14.35 GeV and long undulator of 112 m, while the European XFEL will be even bigger. They may be too expensive to be common. A natural question is: how compact can an XFEL facility be? We have seen in this paper that it is possible to generate a 1.5 Å hard X-ray FEL with lower electron energy (down to 4.5 GeV) and shorter undulator at the expense of a reduced degree of transverse coherence. The radiation power is high enough even with power reduction owing to the undulator wakefield effect. The facility size can be reduced even further by utilizing the third-harmonic radiation, whose power is less than 2% of the fundamental one and transverse coherence is also poor. In conclusion, we cannot build a compact hard X-ray FEL that has all three special properties. However, an XFEL with incomplete transverse coherence is still very useful, because the majority of experiments do not require transverse coherence.

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