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Elaborations on Mössbauer rotor experiments with synchrotron radiation and with usual resonant sources

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A comparative analysis of Mössbauer experiments in a rotating system between a recent application using synchrotron radiation [Friedman *et al.* (2016). *Eur. Phys. Lett.* **114**, 50010; Friedman *et al.* (2017). *J. Synchrotron Rad.* **24**, 661–666] and usual sources of resonant radiation is carried out. The principal methodological difference between these experiments can be related to the fact that in the former set of experiments the source of the resonant radiation rests in a laboratory frame whereas for the latter set of experiments the source is attached to a rotating system. It is concluded that the utilization of ordinary Mössbauer sources remains the most promising path for further research appertaining to the Mössbauer effect in rotating systems.

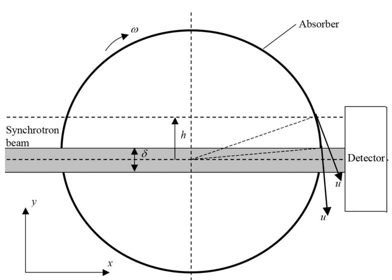
1. Introduction

It is known that the majority of Mössbauer experiments in a rotating system had been carried out by the early 1960s, soon after the discovery of the Mössbauer effect (Hay *et al.*. 1960; Champeney & Moon, 1961; Hay, 1962; Granshaw & Hay, 1963; Champeney *et al.*, 1965; Kündig, 1963). In these experiments either an absorber orbited around a source of resonant radiation or *vice versa*. The goal was to verify the relativistic time dilation for a moving resonant absorber, which induces a relative energy shift between emission and absorption lines by the widely acclaimed value

$$\Delta E/E = -u^2/2c^2, \quad (1)$$

where u is the tangential velocity of the absorber and c is the velocity of light in a vacuum. For sub-sound speeds of $u \simeq$ 300 m s^{-1}, the value of $\Delta E/E$ is of the order of 10^{-12}, which can be reliably measured using iron-57 Mössbauer spectroscopy, because this provides a relative energy resolution of resonant γ-quanta of about 10^{-14} and higher. Correspondingly, all the authors of the mentioned papers reported a confirmation of the classical relativistic expression (1) with an accuracy of about 1%. Later, the relativistic time dilation had been tested – though not in rotation but in uniform translational motion – with a much better precision (10^{-8}–10^{-9}) in experiments on ion beams and free muons undergoing an inertial motion (Bailey *et al.*, 1977; McGowan *et al.*, 1993), whereafter these achievements deprived scientists of further interest in the repetition of such-like Mössbauer rotor setups.

Nonetheless, we recently paid closer attention to the experiments in question in order to verify the prediction by Yarman *et al.* on the additional variation of the time rate in bound systems (*e.g.* Yarman, 2004, 2006). Initially, we devoted



our focus on the renowned experiment conducted by Kündig (1963), since Kündig was the only one to apply a first-order Doppler modulation of energy of γ-quanta on a special rotor setup which provided an oscillating motion of the source along the radius of the rotor, and which hence facilitated the accumulation of Mössbauer spectra of the resonant absorber at each fixed angular velocity. This allowed Kündig to record the shape and the position of the resonant line on the energy scale *versus* the rotational speed. As it so happens, if the vibrations are random, they do not affect the position of the resonant line, so that the measured energy shift between emission and absorption resonant lines becomes practically insensitive to the presence of such mechanical vibrations in the rotor system.

In contrast, others (Hay *et al.*, 1960; Champeney & Moon, 1961; Hay, 1962; Granshaw & Hay, 1963; Champeney *et al.*, 1965) measured only the count rate of detected γ-quanta at each fixed angular velocity, and their results were not protected from the distortions of resonant lines due to vibrations. This explains why Kündig's experiment is much more informative and reliable than the other experiments undertaken on this subject matter. Even so, we regrettably revealed a number of errors in Kündig's data processing calculations (Kholmetskii *et al.*, 2008). The raw data available in the paper by Kündig (1963) allowed us to derive the correct experimental values for the relative energy shift between emission and absorption lines *versus* the rotational frequency, which manifested as

$$\Delta E/E = -ku^2/c^2, \quad (2)$$

with

$$k = 0.596 \pm 0.006. \quad (3)$$

Thus, this result substantially differs from the relativistic prediction k = 1/2, and the revealed difference exceeds the measurement uncertainty by more than ten times.

Moreover, we paid attention to a possible systematic error in the experiment by Kündig (1963), where parts of the piezotransducer – used to implement an oscillating motion to the resonant source – inevitably experience some centrifugal force, which ought to have led to the dependence of the piezoelectric constant on the rotational frequency. Although Kündig estimated this effect to be negligible, a convincing proof was not presented. At the same time, the presence of this effect can decrease the measured energy shift between emission and absorption lines, so that the actual value of the coefficient k must turn out to be even larger in comparison with our rectified estimation (3). Hence, instead of the equality (3), we ended up with the inequality

$$k \geq 0.60. \quad (4)$$

In addition, Kündig observed an approximately exponential increase of the linewidth by up to 1.5 times under a variation of the rotational frequency from 183 rev s^{-1} to 517 rev s^{-1}. Yet this does not mean that the same appreciable increase of linewidth took place for the rotor setups applied in the experiments described in the papers by Hay *et al.* (1960), Champeney & Moon (1961), Hay (1962), Granshaw & Hay (1963) and Champeney *et al.* (1965). At the same time, it is rather difficult to believe that a line broadening was totally absent – as was tacitly assumed by the authors of the above-mentioned papers. Among these, the experiment by Champeney *et al.* (1965) attracts attention on the basis of large amounts of experimental data obtained for different absorbers and the Mössbauer sources ^{57}Co in two different matrices. However, the re-analysis of this experiment, presented by Kholmetskii *et al.* (2008), has shown that, unlike what Champeney *et al.* had originally reported, their outcome too fits well the inequality (4).

The result (4) indicates that, next to the usual relativistic dilation of time for an orbiting resonant absorber, there is an additional component of time dilation arising in a rotating system, which induces an excess of $\Delta E/E$ in comparison with equation (1) to a value much larger than the measured uncertainty.

This finding stimulated the performance of our own experiments on the given subject, though we did not apply a straight repetition of the experiment by Kündig so as to avoid the presence of possible systematic errors in the measurement of the energy shift between emission and absorption lines. Instead, and similarly to what had been effectuated with regards to the experiments reported by Hay *et al.* (1960), Champeney & Moon (1961), Hay (1962), Granshaw & Hay (1963) and Champeney *et al.* (1965), we have decided to measure the count rate of resonant γ-quanta by a detector at different rotational frequencies of the resonant absorber. On the other hand, in contrast to the experiments of Hay *et al.* (1960), Champeney & Moon (1961), Hay (1962), Granshaw & Hay (1963) and Champeney *et al.* (1965), we did evaluate the level of vibrations in the rotor system *via* the measurement of absorption curves for two different resonant absorbers, whose resonant lines are shifted on the energy scale with respect to each other approximately by their linewidth. The idea behind this method is based on the fact that the vibrations do broaden the resonant line but do not affect its area. In addition, if we assume that the vibrations are *random*, then, as we have mentioned above, they do not affect the position of the resonant line on the energy scale. Under these conditions, one can easily realize that an equal broadening of the lines of such absorbers due to vibrations implies a different level of resonant absorption with the change of rotational frequency. Therefore, carrying out a joint processing of the measured data obtained with both resonant absorbers, we were capable of separating the variation of the detector's count rate caused by the energy shift (1) from the variation of the count rate due to vibrations, and in such a way to exclude the influence of vibrations on the measured value of k. The data processing algorithm for the realization of this method presented, for example, by Kholmetskii *et al.* (2009), allows an unbiased estimation of k to be obtained, where the contribution of all possible instrumental factors are explicitly accounted for.

Applying this methodology, our team performed two experiments: first in 2008 (Kholmetskii *et al.*, 2009, 2011) and later in 2014 (Kholmetskii *et al.*, 2015*a*; Yarman *et al.*, 2016*a*), which differ from each other by the technical characteristics of the rotor systems. Accordingly, we arrived at the following results,

$$k = 0.66 \pm 0.03 \qquad (5)$$

(Kholmetskii *et al.* (2009, 2011),

$$k = 0.69 \pm 0.02 \qquad (6)$$

(Kholmetskii *et al.*, 2015*a*; Yarman *et al.*, 2016*a*).

The obtained results (4)–(6) are extraordinary and definitely require an explanation. To date, there have been three attempts to clarify them:

(1) A venture by Corda (2015, 2016) to introduce, under the framework of the general theory of relativity, an additional effect of clock synchronization between the spinning resonant source and the stationary detector placed outside of the rotor system. In particular, the author claims that for this measurement configuration there appears an additional energy shift between the source and the detector with the relative value $(\Delta E/E)_{\rm synch} = u^2/6c^2$, which should be added to the shift due to the time dilation effect $(\Delta E/E)_{\rm dilation} = u^2/2c^2$. Correspondingly, the coefficient k in equation (2) should be determined *via* the summation of $(\Delta E/E)_{\rm synch}$ and $(\Delta E/E)_{\rm dilation}$, which yields $k = 2/3$, in perfect agreement with equations (5) and (6). However, we had already pointed out (Kholmetskii *et al.*, 2015*b*) that the component of energy shift $(\Delta E/E)_{\rm dilation}$ emerges between the source and the absorber, whereas the component of energy shift $(\Delta E/E)_{\rm synch}$ exists between the source and the detector. At the same time, for usual detectors of resonant γ-quanta (used in all Mössbauer rotor experiments performed up to date), their energy resolution should be increased by about ten (!) orders of magnitude, in order to be sensitive to the shift $(\Delta E/E)_{\rm synch} = u^2/6c^2$. Therefore, this component of energy shift remains strictly not tracked by the detector (which only counts resonant γ-quanta, passing across the resonant absorber) and should be totally ignored in the evaluation of k. Further attempts by Corda to defend his approach, *via* the claim (Corda, 2016) that an observer attached to the orbiting absorber and an observer attached to the resting detector measure different energy shifts of resonant lines, directly contradicts classical causality, as we have shown (Kholmetskii *et al.*, 2016).

Thus, the origin of the extra-energy shift between emission and absorption lines in rotating systems still awaits a consistent explanation in the general theory of relativity.

(2) The explanation suggested by Friedman *et al.* on the basis of their generalization of special relativity (Friedman & Semon, 2005; Friedman & Gofman, 2010; Friedman, 2011) with the negation of the 'clock hypothesis' by Einstein (1953). They assumed the presence of a hypothetical maximal acceleration in nature denoted by $a_{\rm m}$, which allegedly affects, in general, the time rate of an orbiting absorber. This then induces an additional energy shift between emission and absorption lines (next to the usual relativistic dilation of time), which, for coefficient k to assume 0.6, leads to the value of $a_{\rm m} \simeq 10^{19}$ m s^{-2}.

(3) The explanation contingent upon the novel framework of the Yarman–Arik–Kholmetskii (YARK) theory of gravity (*e.g.* Yarman & Kholmetskii, 2013; Yarman *et al.*, 2015, 2016*b*), which had, for the past few years, been developed on the basis of Yarman's earlier approach appertaining to any ponderable gravitational interaction scenario (Yarman, 2004, 2006). In the case of the Mössbauer rotor experiments, this theory predicts the exact equality $k = 2/3$ (Yarman *et al.*, 2015), which agrees with the results of the latest measurements (3) and (4).

Now, the entire problem remains open to scientific debate, and new Mössbauer experiments in a rotating system with increased precision are required. In this respect, recent papers by Friedman *et al.* (2016, 2017) that report the results of the application, apparently for the first time, of synchrotron radiation to measure the Mössbauer effect in a rotating system look to be topical.

Friedman *et al.* emphasized that the principal advantage of their experiments, in comparison with the previous Mössbauer rotor experiments where ordinary sources of resonant radiation had been applied (Hay *et al.*, 1960; Champeney & Moon, 1961; Hay, 1962; Granshaw & Hay, 1963; Champeney *et al.*, 1965; Kholmetskii *et al.*, 2009, 2011, 2015*a*; Yarman *et al.*, 2016*a*), is the possibility to measure the entire Mössbauer spectra of an orbiting resonant absorber at different tangential velocities, just like in Kündig's experiment (Kündig, 1963) but in a simpler technical way. Therefore, in the opinion of Friedman *et al.*, much more detailed information can be obtained about the energy shift between a resting source and a rotating absorber, as well as with respect to the influence of various instrumental factors on the measurement results. At the same time, Friedman *et al.* still did not consider the goal of estimating the coefficient k in equation (2); in particular, the paper by Friedman *et al.* (2016) has a methodological character and focuses on the estimation of distortions in Mössbauer measurements caused, first of all, by vibrations in the rotor system.

Based on this analysis, Friedman *et al.* present their recommendations for the improved performance of Mössbauer experiments in a rotating system, which are considered to be universal, *i.e.* equally applicable to both kinds of measurements, either with synchrotron radiation or with ordinary sources. Based on their recommendations, Friedman *et al.* expressed their doubts on the correctness of our results, equations (5) and (6), implying that a further increase of measurement precision should be closely linked to progress in the performance of Mössbauer rotor experiments with a synchrotron source of resonant radiation.

However, we now highlight two principal facts that make an essential difference for the methodologies of both kinds of Mössbauer rotor experiments:

(i) For a synchrotron source which *rests in a laboratory frame*, the linear Doppler effect between the source and the rotating absorber turns out to be essential even for a very narrow synchrotron beam (a few micrometres in width); whereas, in contrast, for an ordinary resonant source which *rests in a rotating frame*, the linear Doppler shift between emission and absorption lines *does not emerge*, and this would be so regardless of the particular configuration of the measurement geometry and divergence of the γ-beam (§2).

(ii) For a synchrotron source, rotor vibrations, *on the whole*, affect the measurement results; whereas, in contradiction, when a resonant source and a resonant absorber are both rigidly fixed on a rotor, it is essential to take into account only *relative vibrations* between them, which, in general, are much smaller in magnitude than the *absolute vibrations* of the rotor itself (§3).

Below, we will show that these facts, when considered together, substantially diminish the practical significance of the methodological recommendations made by Friedman *et al.* (2016) with regards to the realization of Mössbauer rotor experiments where ordinary sources are utilized. We finally conclude in §4 with the conviction that the latter kind of experiments still remain the most promising path for future research on the Mössbauer effect in rotating systems.

2. Mössbauer rotor experiments and the linear Doppler effect

In this section, we consider separately the configuration of Mössbauer rotor experiments with an ordinary point-like source of resonant radiation and with a synchrotron source, and determine the contribution of the linear Doppler effect to the energy shift between emission and absorption lines.

2.1. Mössbauer rotor experiments with usual sources of resonant radiation

In this configuration, a source of resonant radiation [^{57}Co for the experiments described by Hay *et al.* (1960), Champeney & Moon (1961), Hay (1962), Granshaw & Hay (1963), Champeney *et al.* (1965), Kündig (1963), Kholmetskii *et al.* (2009, 2011, 2015*a*) and Yarman *et al.* (2016*a*)] is rigidly attached to the rotor and spins on its axis. Let us show that, in the idealized case where no vibrations are present in the rotor system, the linear Doppler effect between the source and the absorber is totally absent irrespective of their sizes and regardless of the divergence of the resonant beam. In other words, the entire energy shift between emission and absorption lines is proportional to the ratio u^2/c^2 in the lowest order in (u/c).

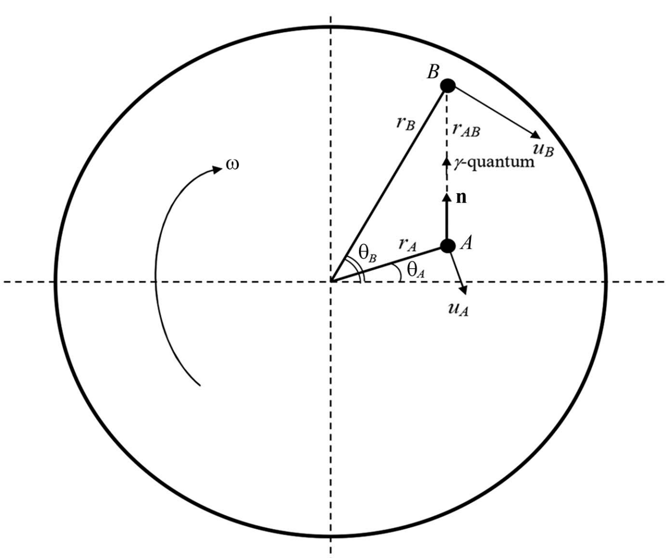


Figure 1
Diagram for the calculation of the frequency (energy) shift between a point-like source A and point-like absorber B, with both items rigidly fixed on the rotor surface.

In order to prove this statement, it is sufficient to show that, for two arbitrary points A and B on the rotor surface (see Fig. 1, where A stands for the point-like source and B for the point-like absorber), the relative energy shift between the emission line and the absorption line does not contain linear terms of the order of (u/c).

For a laboratory observer, the frequency of emitted γ-quanta is equal to (Møller, 1973)

$$\nu_{\rm em} = \frac{\nu_0 \left(1 - u_A^2/c^2\right)^{1/2}}{\left[1 - (\mathbf{n}\cdot\mathbf{u}_A)/c\right]},$$

where ν_0 is the proper frequency of the γ-quanta, $\mathbf{u}_A$ is the tangential velocity of point A at the emission time moment, and $\mathbf{n}$ is the unit vector along the direction of propagation of the γ-quanta as they are emitted from point A and absorbed at point B.

Correspondingly, the frequency of the absorbed radiation in the laboratory frame is

$$\nu_{\rm ab} = \frac{\nu_{\rm em}\left[1 - (\mathbf{n}\cdot\mathbf{u}_B)/c\right]}{\left(1 - u_B^2/c^2\right)^{1/2}} = \frac{\nu_0\left(1 - u_A^2/c^2\right)^{1/2}\left[1 - (\mathbf{n}\cdot\mathbf{u}_B)/c\right]}{\left(1 - u_B^2/c^2\right)^{1/2}\left[1 - (\mathbf{n}\cdot\mathbf{u}_A)/c\right]}, \qquad (7)$$

where $\mathbf{u}_B$ is the tangential velocity of point B at the absorption time moment.

In order to calculate the frequency (7), we designate r_A and θ_A as the radial and angular coordinates of the point A at the moment of emission of a γ-quantum, and, correspondingly, r_B and θ_B as the radial and angular coordinates of point B at the moment of absorption of the γ-quantum the way it can be seen in Fig. 1. With these designations, we have the following components,

$$n_x = \frac{(r_{AB})_x}{r_{AB}} = \frac{r_B\cos\theta_B - r_A\cos\theta_A}{r_{AB}}, \qquad (8b)$$

$$n_y = \frac{(r_{AB})_y}{r_{AB}} = \frac{r_B\sin\theta_B - r_A\sin\theta_A}{r_{AB}}, \qquad (8b)$$

$$u_{Bx} = \omega r_B \sin\theta_B, \quad (9a), \qquad u_{By} = -\omega r_B\cos\theta_B, \quad (9b)$$

$$u_{Ax} = \omega r_A \sin\theta_A, \quad (9c), \qquad u_{Ay} = -\omega r_A\cos\theta_A, \quad (9d)$$

where r_{AB} is the distance between point A at the instant of emission and point B at the instant of absorption.

Hence, substituting (8) and (9) into (7), we derive

$$\begin{aligned}\nu_{\rm ab} &= \frac{\nu_0\left(1-u_A^2/c^2\right)^{1/2}}{\left(1-u_B^2/c^2\right)^{1/2}}\frac{\left[1-(\mathbf{n}\cdot\mathbf{u}_B)/c\right]}{\left[1-(\mathbf{n}\cdot\mathbf{u}_A)/c\right]} \\ &= \frac{\nu_0\left(1-u_A^2/c^2\right)^{1/2}}{\left(1-u_B^2/c^2\right)^{1/2}}\frac{\left[1-\left(n_x u_{Bx}+n_y u_{By}\right)/c\right]}{\left[1-\left(n_x u_{Ax}+n_y u_{Ay}\right)/c\right]} \\ &= \frac{\nu_0\left(1-u_A^2/c^2\right)^{1/2}\left\{1-\left[\omega r_A r_B \sin(\theta_A-\theta_B)\right]/r_{AB}\,c\right\}}{\left(1-u_B^2/c^2\right)^{1/2}\left\{1-\left[\omega r_B r_A \sin(\theta_A-\theta_B)\right]/r_{AB}\,c\right\}} \\ &= \frac{\nu_0\left(1-u_A^2/c^2\right)^{1/2}}{\left(1-u_B^2/c^2\right)^{1/2}}. \end{aligned} \quad (10)$$

Thus, the terms of the numerator and the denominator, which contain linear terms in (u/c), mutually cancel each other, so that *no contribution* of the linear Doppler effect to the energy shift between emission and absorption lines emerges. Therefore, the frequency (energy) shift is determined by the second-order Doppler shift (or time dilation effect) alone, as long as the extra energy shift revealed in the experiments presented by Kholmetskii *et al.* (2009, 2011, 2015*a*) and Yarman *et al.* (2016*a*) is not taken into account at this time.

Since equation (10) has been derived for two arbitrary points A and B on the rotor surface, it also remains in force for any spatially extended source and absorber fixed on the rotor surface, and does not depend on the divergence of the γ-beam. The only point is that, for such a spatially extended source centered on the rotational axis, the tangential velocity u_A at the edge of the source and the tangential velocity at the center evidently differ from each other, and this can cause a broadening of the emitting resonance line. However, for a source of resonant radiation sufficiently compact and bearing a typical configuration of customary Mössbauer rotor experiments, this effect is quite negligible as numerically estimated by Kündig (1963). Thus, for practical purposes, we can put $u_A = 0$ in the case of a compact source, so that the relative energy shift becomes

$$\frac{\Delta E}{E} = \frac{\nu_0-\nu_{\rm ab}}{\nu_0} = 1-\frac{1}{\left(1-u_B^2/c^2\right)^{1/2}} \simeq -\frac{u_B^2}{2c^2} \quad (11)$$

[where we temporarily exempt ourselves from dealing with the extra energy shift revealed in experiments detailed by Kholmetskii *et al.* (2009, 2011, 2015*a*) and Yarman *et al.* (2016*a*)].

Therefore, we conclude that the broadening of the resonant line observed in Kündig's experiment (and definitely taking place in all other experiments that feature an ordinary resonant source fixed on a rotor system) should be attributed to rotor vibrations only, as Kündig (1963) assumed.

2.2. Mössbauer rotor experiment with a synchrotron source

In this section, we will consider the experiments presented by Friedman *et al.* (2016, 2017), where a source of synchrotron radiation is applied to measure the energy shift of the resonant line in an orbiting absorber. In these experiments, the authors used a rotor with a 50 mm radius capable of rotating with a frequency of up to 1 kHz. A semicircular-shaped single-line absorber [an enriched ^{57}Fe(95%)$K_4Fe(CN)_6 \cdot 3H_2O$ single-line material] was placed on the rim of the disk, and the synchrotron Mössbauer source (SMS) of the European Synchrotron Radiation Facility (providing ^{57}Fe resonant radiation at 14.4 keV within a bandwidth of 15 neV) was focused by Kirkpatrick–Baez optics to a 10 µm × 5 µm spot size. Hence, the beam from the SMS first hits the rotating absorber, whereafter it is detected by the detector diametrically opposed to the SMS. This beam crosses the rotor along its diameter. During rotation, the radial acceleration of the absorber alternately assumes parallel [denoted as state (*b*)] and anti-parallel [denoted as state (*a*)] orientations to the photon wavevector [see Fig. 1 of Friedman *et al.* (2016)]. According to Friedman *et al.*, this can yield opposite signs for the additional frequency shift due to the aforesaid acceleration as postulated by Friedman & Semon (2005), Friedman & Gofman (2010) and Friedman (2011). There are some technical improvements in the latest experiment (Friedman *et al.*, 2017) in comparison with the earlier experiment (Friedman *et al.*, 2016), which, however, do not affect the principal methodology of both experiments and thus are not commented on here; separate attention to the experiment (Friedman *et al.*, 2017) is given in our recent paper (Kholmetskii *et al.*, 2018).

Further, we emphasize that, in the experiments by Friedman *et al.*, the source of resonant radiation rests in the laboratory frame; and hence the contribution of the linear Doppler effect to the energy shift between resonant lines of the source and the rotating absorber does not, in general, vanish. In particular, one can realize that the finite projection of the tangential velocity of the resonant absorber onto the axis of the synchrotron beam emerges due to two circumstances: the finite width of the synchrotron beam, denoted hereinafter as δ (see Fig. 2), and the finite distance between the synchrotron beam and the axis x of Fig. 2, denoted hereinafter as h. Due to the finite values of δ and h, the expression

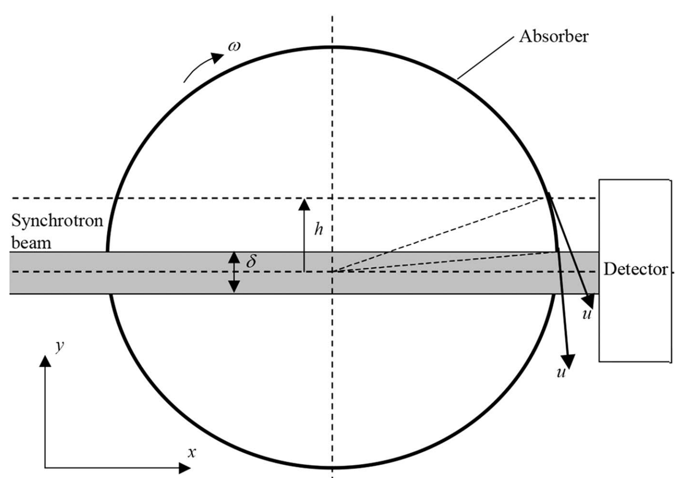


Figure 2
The linear Doppler effect contribution due to the final width δ of the synchrotron beam, and also due to the finite distance h between the axis x and the axis of the synchrotron beam. In both cases the tangential velocity of the resonant absorber has a finite projection onto the axis of the synchrotron beam.

for the energy shift between emission and absorption lines should contain linear terms in (u/c), which are responsible for the corresponding linear Doppler effect contribution.

The finite value of the width of the synchrotron beam δ leads to the broadening of the resonance line, insofar as γ-quanta on the periphery of the beam pass the point of the resonant absorber where the absorber's tangential velocity has a finite projection on the propagation direction of such γ-quanta (see Fig. 2), with the tangential velocity being equal to

$$\delta u \simeq (\delta/2r)\,u = \delta\omega/2, \tag{12}$$

where r is the rotor radius and ω is the angular velocity. Hence, at δ = 5 µm and given the rotational frequency ν = 300 rev s^{-1} [the maximum value used in the experiment (Friedman *et al.*, 2016), corresponding to the tangential velocity of the absorber of about 100 m s^{-1}], equation (12) yields

$$\delta u \simeq 4.7 \text{ mm s}^{-1}, \tag{13}$$

which substantially exceeds the proper width of the resonant line (about 0.3 mm s^{-1} for ^{57}Fe).

Therefore, at the maximal tangential velocity near 300 m s^{-1} (applied in the experiments with ordinary sources performed up to now), the width of the resonant line would be three times larger, *i.e.* about 14 mm s^{-1}, which exceeds the natural linewidth by approximately 50 times! Thus, the sensitivity to a relative energy shift between emission and absorption lines decreases by the same amount of times in comparison with rotor experiments using ordinary sources, even in the idealized case where any vibrations in the rotor are fully suppressed.

The Mössbauer spectrum of the resonant absorber measured at ν = 300 rev s^{-1} ($u \simeq$ 100 m s^{-1}) is shown in the lower part of Fig. 3 of the paper by Friedman *et al.* (2016). Though Friedman *et al.* do not explicitly indicate the width of their resonant line, it can be qualitatively estimated to be near 9–10 mm s^{-1}, which approximately twice exceeds the value (13). Thus, we can conclude that the observed broadening of the resonant line happens not only due to the final width of the synchrotron beam but due to vibrations too. Unfortunately, the authors (Friedman *et al.*, 2016) do not present any estimation of the relative contribution of these effects to the measured linewidth. Under these conditions it is reasonable to assume that, at the given rotational frequency ν = 300 rev s^{-1} (corresponding to u = 100 m s^{-1}), the contribution of rotor vibrations to the line broadening has approximately the same value (13), and its further analysis is given in the next section.

Further, when the synchrotron beam as a whole is shifted a distance h along the axis y of Fig. 2, this induces a linear Doppler shift of the measured resonant line of the orbiting absorber, which is named by Friedman *et al.* as the *alignment shift* (AS). In this case, similar to equation (13), the projection of the tangential velocity of the absorber to the propagation direction of the γ-quantum at the point of contact is defined at a small h as

$$\Delta u_{\rm AS} \simeq (h/r)\,u = h\omega. \tag{14}$$

By the same token, Friedman *et al.* (2016, 2017) find important the fact that the *alignment shift* has *the same values* in the state (a) (when the radial acceleration of the absorber is anti-parallel to the photon wavevector) and in the state (b) (when the radial acceleration of the absorber is parallel to the photon wavevector); and hence the *relative AS* between these states, in the idealized case of the absence of any vibrations, should vanish, so that the influence of acceleration on the relative energy shift of the resonant line can be investigated in a pure form.

In a real situation, the vibrations in the rotor system are always present, and, in the case where they are not random (and which, in general, is actually the case), the *relative AS* between the states (a) and (b) is not vanishing (see §3).

3. Mössbauer rotor experiments and vibrations

In the general case, vibrations in the rotor system lead to the broadening of the resonant line; in addition, when a non-random component of such vibrations is present, it may affect the position of the resonant line on the energy scale. It is then obvious that, for the case where a source of resonant radiation rests in a laboratory frame, like in the Mössbauer rotor experiments with synchrotron radiation (Friedman *et al.*, 2016, 2017), absolute rotor vibrations, which are measured in the laboratory frame, directly affect the resonant absorption. Contrariwise, where the source of resonant radiation is rigidly fixed to the rotor (like for Mössbauer rotor experiments with ordinary resonant sources), only relative vibrations between the source and the absorber may distort the resonant line. It is obvious that, under any reasonable experimental conditions, the relative vibrations are always less than the absolute vibrations of the rotor, and, thus, from this point of view, the experiments with ordinary sources look much more attractive than the experiments with synchrotron radiation.

Nevertheless, the analysis of the influence of *absolute* rotor vibrations on the measurement results implemented by Friedman *et al.* (2016) in the case of synchrotron radiation is still interesting. Carrying out this analysis, Friedman *et al.* applied the Jeffcott model for the description of rotor vibrations, and determined a non-random component throughout such vibrations. They anticipated that the presence of this non-random component induces an additional energy shift between emission and absorption lines, which is comparable with the proper width of the resonant line. Thereby, Friedman *et al.* concluded that the results (5) and (6) obtained by our team are not convincing, because our data processing procedure described in Kholmetskii *et al.* (2009, 2011, 2015*a*) and Yarman *et al.* (2016*a*) implies that the vibrations are *random*, and apparently remains in contrast with the observations by Friedman *et al.* (2016).

However, Friedman *et al.* did not realize that in these experiments (Kholmetskii *et al.*, 2009, 2011, 2015*a*; Yarman *et al.*, 2016*a*), which led to the results (5) and (6), the *relative* vibrations between the source and the absorber are indeed much smaller than the absolute rotor vibrations observed in their measurements; thus, the results obtained by Friedman *et* *al.* (2016) cannot be straightforwardly extended to the case of the application of a usual source mounted on a rotor.

In the latter case, the relative vibrations between the source and the absorber induce a fluctuation of their relative velocity, which is defined as the difference – as seen by the laboratory observer – between the velocity of the source at the emission moment and the velocity of the absorber at the absorption moment. It is then obvious that, in being responsible for the linear component of the Doppler shift between emission and absorption lines, only the *radial component* of such relative velocity should be taken into account. In order to estimate such a radial component of relative velocity, we point out that the displacement of the rotor *on the whole* due to its vibration along the line joining the source and the absorber at the value δx (where we assume that this line is parallel to the x-axis at the considered time moment) during a short time interval δt induces a much smaller *relative* displacement δx_{sa} between the source and the absorber, which is determined *via* the speed of sound v_s in the rotor material, *i.e.*

$$\delta x_{sa} \simeq \delta x \sin(\omega r / v_s). \quad (15)$$

In this equation we adopted the common fact that the main harmonic of the vibration frequency spectrum coincides with the rotational frequency. At $\omega r = 100$ m s^{-1} [the maximal tangential velocity of the absorber in the experiment (Friedman *et al.*, 2016)] and $v_s \simeq 6000$ m s^{-1} (speed of sound in an aluminium alloy), the latter equation yields

$$\delta x_{sa} \simeq \delta x / 60. \quad (16)$$

The same relationship (16) holds for the radial component of *relative velocity* between the source and the absorber caused by rotor vibrations, and, thus, for known configurations of the Mössbauer rotor experiments with usual sources, the influence of rotor vibrations on the shape of the resonant line is almost two orders of magnitude smaller than that framed by the synchrotron experiment reported by Friedman *et al.* (2016). Therefore, the systematic component of the energy shift between emission and absorption lines due to vibrations estimated in the experiment by Friedman *et al.* (2016) (amounting to about 0.60 mm s^{-1}) is reduced to the value $0.60/60 = 1 \times 10^{-2}$ mm s^{-1}. This value is a few tens of times smaller than the proper width of the resonant line, and is quite negligible for Mössbauer rotor experiments using ordinary sources. Hence, the assumption about the random character of vibrations for the kind of experiments undertaken by Kündig and reported by Kündig (1963) as well as by other authors of Mössbauer rotor experiments using ordinary resonant sources (Hay *et al.*, 1960; Champeney & Moon, 1961; Hay, 1962; Granshaw & Hay, 1963; Champeney *et al.*, 1965; Kholmetskii *et al.*, 2009, 2011, 2015*a*; Yarman *et al.*, 2016*a*) is quite warranted in any practical situation, and makes fully reliable our recent results (5), (6) which were obtained on the basis of the given assumption.

Therefore, the explanation of the origin of the extra-energy shift between emission and absorption lines for Mössbauer experiments in a rotating system, as expressed by equations (5) and (6), remains in force as a very topical problem.

Looking closer at the contribution of vibrations to the measured shift of the resonant line in the experiment conducted by Friedman *et al.* (2016), we recall the disclosure of the previous section where we found that the contribution of rotor vibrations to the line broadening is comparable with the value (13) at u = 100 m s^{-1}. Insofar as the level of vibrations is proportional to u^2, then at $u \simeq 300$ m s^{-1} (a typical maximal tangential velocity in Mössbauer experiments with ordinary sources) the line broadening due to vibrations is expected to be nine times larger in comparison with the value (13), *i.e.* more than 40 mm s^{-1}. This exceeds the natural width of the resonant line ($\leq$0.3 mm s^{-1}) by more than 100 times (!), which, in fact, leads to a drastic decrease of measurement sensitivity to the relative energy shifts between emission and absorption lines under the rotation of the absorber. This means that the uncertainty in the determination of the coefficient k in equation (2) in the Mössbauer rotor experiments with synchrotron radiation should many times exceed the measurement uncertainty of the coefficient k in the experiments with ordinary sources of resonant radiation. This circumstance explains the fact that Friedman *et al.* did not even try to evaluate the coefficient k in both experiments (Friedman *et al.*, 2016, 2017).

Thus, Friedman *et al.* (2016) restricted their research to the measurement of the difference in AS between the state (a) (when the radial acceleration of the absorber is anti-parallel to the photon wavevector) and the state (b) (when the radial acceleration of the absorber is parallel to the photon wavevector). They claimed that, for run 39 of Friedman *et al.* (2916), the observed relative shift of the line between states (a) and (b) 'is significantly larger than the calculated vibrational one'. In such a way, the authors presume to hint that their result indicates a violation of the clock hypothesis by Einstein, and that we are dealing with the presence of a maximal acceleration in nature.

However, their results are not convincing because the shift of the resonant line due to the non-random character of vibrations in the rotor system does not provide quantitative information about the corresponding relative AS. The reason is that the line broadening in the presence of vibrations is defined by the fluctuation of the velocity of the absorber along the synchrotron beam (the axis x in Fig. 2), whereas the fluctuation of the AS is defined by the fluctuation of the distance h due to vibrations, which, in general, has different values in the states (a) and (b) for non-random vibrations, and cannot be estimated *via* the line broadening. The same remark remains in force with respect to the latest experiment by Friedman *et al.* (2017) (see Kholmetskii *et al.*, 2018).

In fact, a correct way to determine the shift of resonant lines as a function of rotational frequency is to measure the coefficient k in equation (2) as was done in all experiments with ordinary sources of resonant radiation performed up to now. However, this is still not the case for the synchrotron experiment reported by Friedman *et al.* (2016, 2017), where low measurement sensitivity to the relative energy shifts of resonant lines due to considerable broadening of the lines still does not allow the coefficient k in equation (2) to be determined with the measurement precision comparable with the values obtained in other experiments on this subject, where ordinary sources of resonant radiation had been applied.

4. Conclusion

As an outcome of this comparative study, we conclude that the quotidian application of compact point-like ordinary sources of resonant radiation rigidly attached to a rotor remains the most efficient way to perform Mössbauer experiments in a rotating system. Experiments on this subject with a synchrotron source (which rests in a laboratory frame) have a number of principal shortcomings in comparison with the case of ordinary sources as disclosed in the present research. Correspondingly, among practical recommendations made in the concluding section of the paper by Friedman *et al.* (2016) with respect to improved performance of Mössbauer experiments in a rotating system, only the initial (and obviously trivial) suggestion to use 'vibrationless rotor systems' is relevant for both kinds of experiments.

All other practical recommendations by Friedman *et al.* (2016), such as, in particular, being mindful of the non-random component of vibrations in the rotor system, are relevant only for further development of Mössbauer rotor experiments relying on a synchrotron source, and are not appreciably significant for experiments making use of ordinary sources; which, due to the absence of the linear Doppler shift of the resonant line, as well as a diminished level of vibrations between the source and the absorber, will remain the most promising path for further research on the Mössbauer effect in rotating systems in the foreseeable future. This fact is all the more accentuated in the case of other plausible Mössbauer setups with variations on the placement of the source and the absorber.

Perspectives for the performance of such experiments with improved precision have been recently analyzed by Kholmetskii *et al.* (2016).

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